Influence of the differential rotation on stability properties of neutron stars

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1 Introduction

Newly born, hot neutron stars (NSs) or a remnant of a binary NS coalescence rotate differentially. Differential rotation can temporarily stabilize a massive star against prompt collapse to a black hole (BH). This may have important consequences for gravitational waves observations and explanations of gamma ray bursts.

In this contribution we present a study on the stability properties of differentially rotating NSs using a modified version of a highly stable, relativistic code *FlatStar* (Ansorg et al., 2009, and references therein). The high level of accuracy of the code enables us to construct various types of configurations which were not considered previously, mainly due to numerical limitations. In particular, for broad ranges of the degree of differential rotation we calculate the dimensionless Kerr parameter J/M^2 (we assume $G = c = 1$), where J and M are the angular momentum and the total gravitational mass of a rotating NS, respectively. It was shown (e.g., Baumgarte et al. 2000 and Giacomazzo et al. 2011) that a differentially rotating NS with the Kerr parameter lower than 1, a *sub-Kerr* NS, is dynamically unstable and collapses promptly to a rotating BH. On the other hand, the stellar models with $J/M^2 > 1$, *supra-Kerr* configurations are dynamically stable and are subject to various secular instabilities. The gravitational wave emission from supra-Kerr models are supposed to be considerably larger than that for sub-Kerr models. The above results were found for several models of differentially rotating NS.

Since exact criteria do not exist for dynamical stability of rotating NSs, the first step is to explore the properties of rotating NSs close to the stability limit of nonrotating NS. A spherical star is dynamically stable (unstable) against radial oscillations, if its central energy density is lower (higher) than the central density of a star with the maximum mass. Numerical simulations for uniformly rotating NSs show that the central densities of configurations marginally stable to the axisymmetric perturbation are lower only by up a few % to 30 % depending on the equation of state (EOS) of dense matter. Following Giacomazzo et al. (2011) we focus on differentially rotating NSs with maximum densities equal to the central density of a spherical NSs with the maximum mass. Giacomazzo et al. (2011) found that all models are sub-Kerr and dynamically unstable. In contrast to the authors we compute the Kerr parameter for all types (Gondek-Rosińska et al., 2014) of differentially rotating NS to verify if we can find supra-Kerr models for this particular maximum density.

2 Assumptions

We construct relativistic axisymmetric models of differentially rotating NSs using a dual-domain pseudo-spectral code (Ansorg et al., 2009). We employ an astrophysically motivated, well-known "j-constant" law of differential rotation introduced by (Komatsu et al., 1989), where inner layers of a star have higher angular velocities Ω than external ones: $F(\Omega) = A^2(\Omega_c - \Omega)$. The constant A represents the length-scale over which the angular velocity changes (the degree of differential rotation), while Ω_c is the angular velocity on the rotation axis. We use rescaled value of A with respect to the equatorial radius R_e: $\tilde{A} = R_e/A$. The higher the $\tilde{A}$, the higher degree of differential rotation and the steeper angular velocity profile is, while for uniform rotation $\tilde{A} \to 0$.

To describe NS we use a polytropic EOS $P = \kappa\rho^\Gamma$, where P is the pressure, ρ is the rest-mass density, κ and Γ are a polytropic constant and a polytropic index respectively. In our paper we consider $\Gamma = 1.8$ (moderately stiff EOS) and $\kappa = 1$.

3 Calculations and results

In order to investigate the effect of the differential rotation on the Kerr parameter we have constructed a large number of rapidly rotating models for a wide range of $\tilde{A}$ (between 0 and 1.5) for fixed maximum energy density, 0.1869, equal to the maximum density of nonrotating NS with the maximum mass. For each value of $\tilde{A}$ we have calculated sequences of stellar models starting with nonrotating star and decreasing the ratio of the polar to the equatorial radius r_{rat}, in small increments until we reach the mass-shedding limit or a configuration with $r_{\text{rat}} = 0$. According to the classifications of Ansorg et al. (2009) we call them **type A** and **type C** sequences. For each sequence we look for the maximum value of the Kerr parameter, which was obtained at the end of the sequence. Type A models exist for $0<\tilde{A}<1$ while type C for $1 \leq \tilde{A} \leq 1.5$. In addition we performed calculations for two new types of differentially rotating NS which have no connection with nonrotating limit: **type B** existing for $0.8 \leq \tilde{A} \leq 1$ and **type D** for $1 \leq \tilde{A} \leq 1.1$.

The maximum of J/M^2 for all types of differentially rotating NSs versus the degree of differential rotation is plotted on Fig. 1. The results obtained by Giacomazzo et al. (2011) for the same equation of state are shown for comparison (the long dashed line). We summarize our results showing that for

- type A - J/M^2 is a increasing function of $\tilde{A}$ (dashed line),
- type C - J/M^2 is a decreasing function of degree of differential rotation (dash-dotted line)
- type B and D - J/M^2 is a decreasing function of $\tilde{A}$ being very close to the threshold value $J/M^2 = 1$ (solid lines).

We see a very good agreement for type A models with Giacomazzo et al. (2011), while for type C we are well above their results since their code failed to the solution for small values of r_{rat}. Our calculations confirm results of Giacomazzo et al. (2011) that all models are sub-Kerr. The highest value of the Kerr parameter, but still lower than 1 is obtained for the new type configurations (B and D), which were not considered before. The dynamical calculations of all types of models are required to find stability criteria for differentially rotating NS. Our accurate results can be

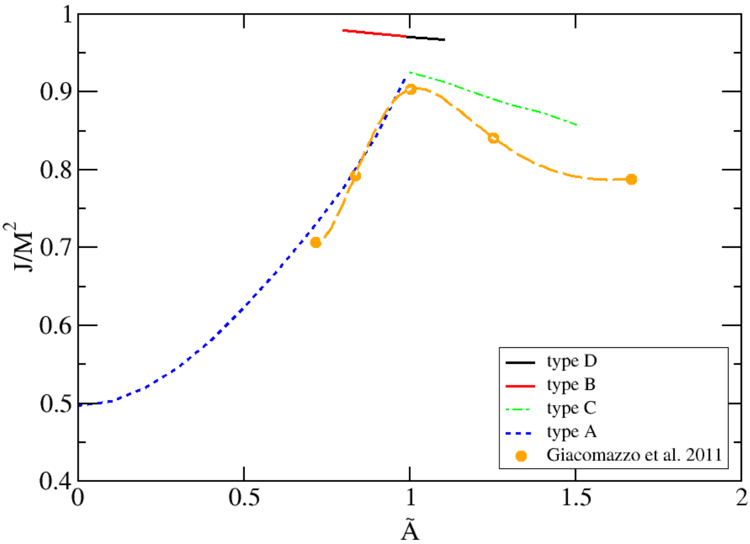

Fig. 1: The Kerr parameter versus degree of differential rotation $\tilde{A}$ for differentially rotating NS described by a polytropic EOS with $\Gamma = 1.8$. All four types of differentially rotating NS are plotted: a dark blue dashed line corresponds to type A, a green dotted-dashed line to type C, and two solid lines to type B and type D models, respectively. Results obtained by Giacomazzo et al. (2011) are plotted as a long dashed line with filled circles.

used as initial data for hydrodynamical simulations. We conclude that the results obtained for type B configurations with the lowest values of A may reach $J/M^2 > 1$ for maximum density close to the one considered by us and the stellar models with a slightly lower maximum density may be stabilized against prompt collapse to a BH. Such stars could be promising sources of gravitational waves for future detectors.

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