# The effect of rotation on the properties of neutron stars

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## 1 Introduction

Neutron stars (NS) are the densest, fastest rotating, and the most stable objects in the Universe, the density in their interior is even higher than the nuclear density. The real form of matter in such high densities is still unknown. There are many hypotheses about realistic equation of state (EoS): from very simple with neutrons, protons, electrons and muons, through more complex with hyperon condensates or superfluidity to the exotic ones with strange quark matter or kaon or pion condensation in the stellar core. Independently of EoS, rotation has an important impact on the properties of NSs. A newly born, hot NS rotates differentially (the closer to the stellar centre the higher spin velocity is). Cold NSs rotate uniformly, since any fluctuations are suppressed by high viscosity of the matter. The rotation frequency is limited by the mass-shedding frequency (the Keplerian limit) when the centrifugal force on the surface is equal to the gravitational force. The theoretical limits on uniform rotation are 0.4 ms to 1.2 ms for different EoS of NSs (Cook et al., 1994) and 0.36 ms to 0.6 ms for strange quark stars (SQS) (Gourgoulhon et al., 1999; Gondek-Rosińska et al., 2000). The fastest observed NS is a pulsar PSR J1748-2446ad, rotating with 1.4 ms (Hessels et al., 2006). The difference between observed frequency and theoretical one can be explained by gravitational wave emission - rotating star could lose its angular momentum by breaking axial symmetry.

Rotation may have important impact on the properties of NS, e.g., it can increase their masses, radii, decrease central density and significantly change their shapes. In the article we summarize the results on the effect of the rotation (uniform and differential) on the properties of NSs (including SQSs). We present the first relativistic, numerical results of differentially rotating SQSs.

## 2 Rotating neutron stars

Using Tolman-Oppenheimer-Volkoff equations for fixed EoS and central density we can determine basic parameters, mass $M$ and radius $R$ of non-rotating NSs. The maximum masses $M_{\max}$ are found to be in range 1.5 - 2.7 $M_\odot$ and corresponding $R$ in the range 7 - 14 km. The maximum radius of non-rotating NSs can be up to 18 - 24 km for NS and up to 11 km for bare SQSs (Cook et al., 1994; Gourgoulhon et al., 1999).

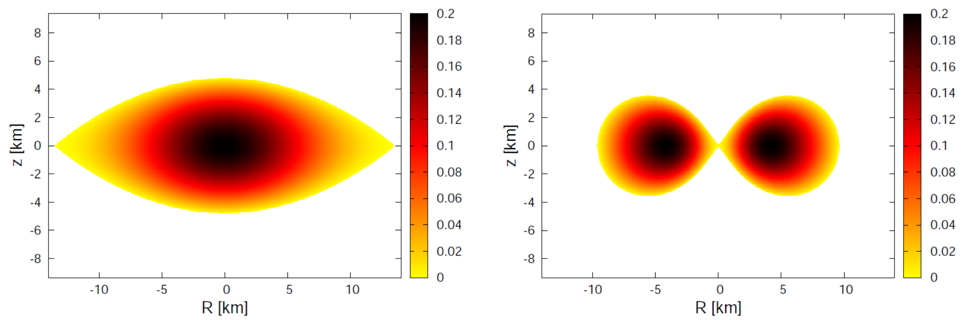


Fig. 1: Cross section of differentially rotating SQS with $\rho_{\rm max} = 8.2 \times 10^{14}$ g/cm$^3$, the colors describe rescaled value of density in the star in range from 0 ($4.2785 \times 10^{14}$ g/cm$^3$) to 0.2 ($8.2 \times 10^{14}$ g/cm$^3$). *Left panel*: Star at Keplerian limit with $\tilde{A} = 0.2$. *Right panel*: Star at toroidal configuration with $r_{\rm pole}/r_{\rm eq} = 0.01$ and $\tilde{A} = 0.5$.

Rigid (uniform) rotation may stabilize stars more massive than $M_{\rm max}$ of static configurations (Cook et al., 1994). For configurations with fixed baryon mass spun up from static limit to the Keplerian limit, the increase of gravitational mass is $\sim 5$ %, but of radius by $\sim 30$ -60 % for NS described by realistic EoS and by $\sim 70$ % for SQS. The maximum allowed mass $M_{\rm max,rot}$ of rigidly rotating NS is, depending on EoS, 14 % - 22 % higher than $M_{\rm max}$ of non-rotating stars (corresponding radius $\sim$ 30 % in most cases) (Cook et al., 1994), for SQS 42 % - 44 % (corresponding radius 50 %) (Gondek-Rosińska et al., 2000; Gourgoulhon et al., 1999).

Fast rotation changes the shape of a NS. The ratio of polar to the equatorial radius, $r_{\rm pole}/r_{\rm eq}$, tells us how 'flattened' the stars is. For rigidly rotating NSs this parameter is greater than 0.55, and for SQSs than 0.34.

Differential rotation has important impact on the structure and properties of newly born NS, when an EoS depends not only on the density but also on the temperature. Low shear viscosity allows the stellar layers rotate with different velocities. The stellar core may rotate faster than the envelope (Komatsu et al., 1989) therefore star could support a significantly larger mass than the uniformly rotating NS with the same baryon mass (Baumgarte et al., 2000). Differential rotation can stabilize NS and support it against prompt collapse into a black hole.

The first systematic relativistic studies of differentially rotating NS with realistic EoS were done by Morrison et al. (2004). They performed calculations for four values of degree of differential rotation $\tilde{A} = 0.3, 0.5, 0.7, 1.0$ (the higher the $\tilde{A}$, the larger the degree of differential rotation, i.e., the higher ratio of the spin velocity in the centre to the spin velocity at the surface at the equatorial radius). We compare their results with our calculations of differentially rotating SQSs (Szkudlarek et al., 2014).

We have performed calculations for five values of $\tilde{A} = 0.1, 0.2, 0.3, 0.5, 0.7$ for rotating SQSs described by MIT Bag model using a modified version of highly accurate relativistic code FlatStar (Ansorg et al., 2009, and references therein). On Fig. 1 we show two examples of differentially rotating SQSs with $M_{\rm max,rot}$ for a fixed degree of differential rotation: $\tilde{A} = 0.2$ (left) and $\tilde{A} = 0.5$ (right). The first one is a configuration close to the Keplerian limit, the second one has a toroidal shape. Both configurations were obtained by calculating a sequence of stars parametrized by $r_{\rm pole}/r_{\rm eq} \to 0$ keeping $\rho_{\rm max}$ and $\tilde{A}$ fixed. We started with $r_{\rm pole}/r_{\rm eq} = 1$ (a non-rotating configuration).

On Fig. 2 (left panel) we show the $M_{\rm max,rot}$ as a function of maximum density

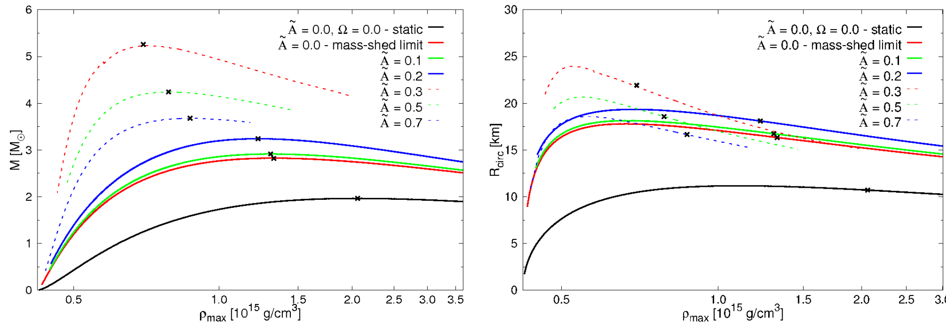


Fig. 2: Gravitational mass $M$ (*left panel*) and circumferential radius $R_{\rm circ}$ (*right panel*) as a function of $\rho_{\max}$. Black solid line represents static stars, red solid line represents rigidly rotating $M_{\max}$ configurations, other lines represent differentially rotating stars (solid lines - sequence ends at Keplerian limit, dashed lines - sequence ends when $r_{\rm pole}/r_{\rm eq} \to 0.0$). For higher values of $\tilde{A}$ maximum mass configurations are reached at $r_{\rm pole}/r_{\rm eq} \sim 0.0$ Configurations with $M_{\max}$ in the sequence are marked with black crosses.

$\rho_{\max}$ for SQS. Each line corresponds to the sequence with fixed $\tilde{A}$. The $R$ versus $\rho_{\max}$ (right panel) shows that the $M_{\rm max,rot}$ configuration does not coincide with $R_{\rm max,rot}$ configuration. This maximum is reached for lower values of mass at lower $\rho_{\max}$.

Increase of the $M_{\rm max,rot}$ for NS is in the range 17 % - 60 %, and for SQS 48 % - 168 %. Corresponding $r_{\rm pole}/r_{\rm eq}$ is in the range 0.01 - 0.5 and 0.01 - 0.45 respectively. Comparison for each $\tilde{A}$ in both simulations looks as follows: for $\tilde{A} = 0.3$ increase of the $M_{\max}$ for NS is 28 % - 40 % (corresponding $r_{\rm pole}/r_{\rm eq}$ 0.43 - 0.5), for SQS 168 % (0.01); for $\tilde{A} = 0.5$ for NS 25 % - 56 % (0.32 - 0.5), for SQS 116 % (0.01); for $\tilde{A} = 0.7$ for NS 19 % - 60 % (0.01 - 0.28), for SQS 87 % (0.01).

We conclude that independently of EoS the $M_{\rm max,rot}$ increases till some critical value of $\tilde{A}$, reaches the maximum and then decreases. The mass grows only for stars which ends close to the Keplerian limit. The maximum of maximal masses is obtained for critical value of $\tilde{A}$ for a star with a toroidal shape. For higher value of $\tilde{A}$ then critical one the maximum mass decreases with increasing $\tilde{A}$. In case of differential rotation the increase of mass is much higher for SQSs than for NS described with realistic EoS considered here.

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