Order and chaos in hydrodynamic BL Her models

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We present the full wealth of dynamic behaviours characteristic to deterministic chaos that we have found in a sequence of hydrodynamic BL Her-type models.

Using Warsaw non-linear convective pulsation codes (Smolec & Moskalik, 2008) we have conducted a hydrodynamic survey of BL Her-type models (fixed $M = 0.55$ M$_\odot$, fixed $L = 136$ L$_\odot$, $X = 0.76$, $Z = 0.0001$). In these models we have decreased the eddy-viscous dissipation. Between the effective temperatures of ≈ 6340 K and ≈ 6520 K the models show a wealth of dynamical behaviours characteristic to chaos. This is best illustrated with the bifurcation diagram plotted in Fig. 1. It is a stack of grey-shaded histograms of possible values of maximum radii during pulsations, plotted versus the model's effective temperature. The well visible chaotic bands are born through a series of period doubling bifurcations both from the cool and from the hot side of the computation domain (so-called period doubling route to chaos). Within chaotic bands windows of periodic, period-k behaviour are detected including e.g. large period-3 (between 6421 K and 6438 K) and period-6 windows (6459 K – 6468 K) and smaller period-5, period-7 or period-9 windows. Detailed analysis of these models, including discussion of their reliability and importance for understanding of more luminous, longer period, semi-regular variables is presented in Smolec & Moskalik (2014). Here we just highlight few interesting phenomena detected for the first time in the models of pulsating stars.

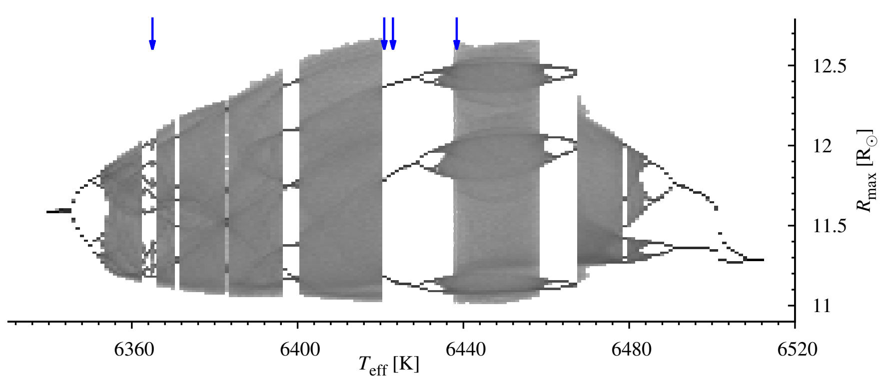

Fig. 1: Bifurcation diagram for the computed hydrodynamic models. Arrows show the location of models presented in Fig. 2.

In Fig. 2 we plot the consecutive values of maximum radii for four interesting models. Location of these models in the bifurcation diagram (Fig. 1) is marked with arrows. In panel a) we observe a periodic behaviour – a period-3 cycle.

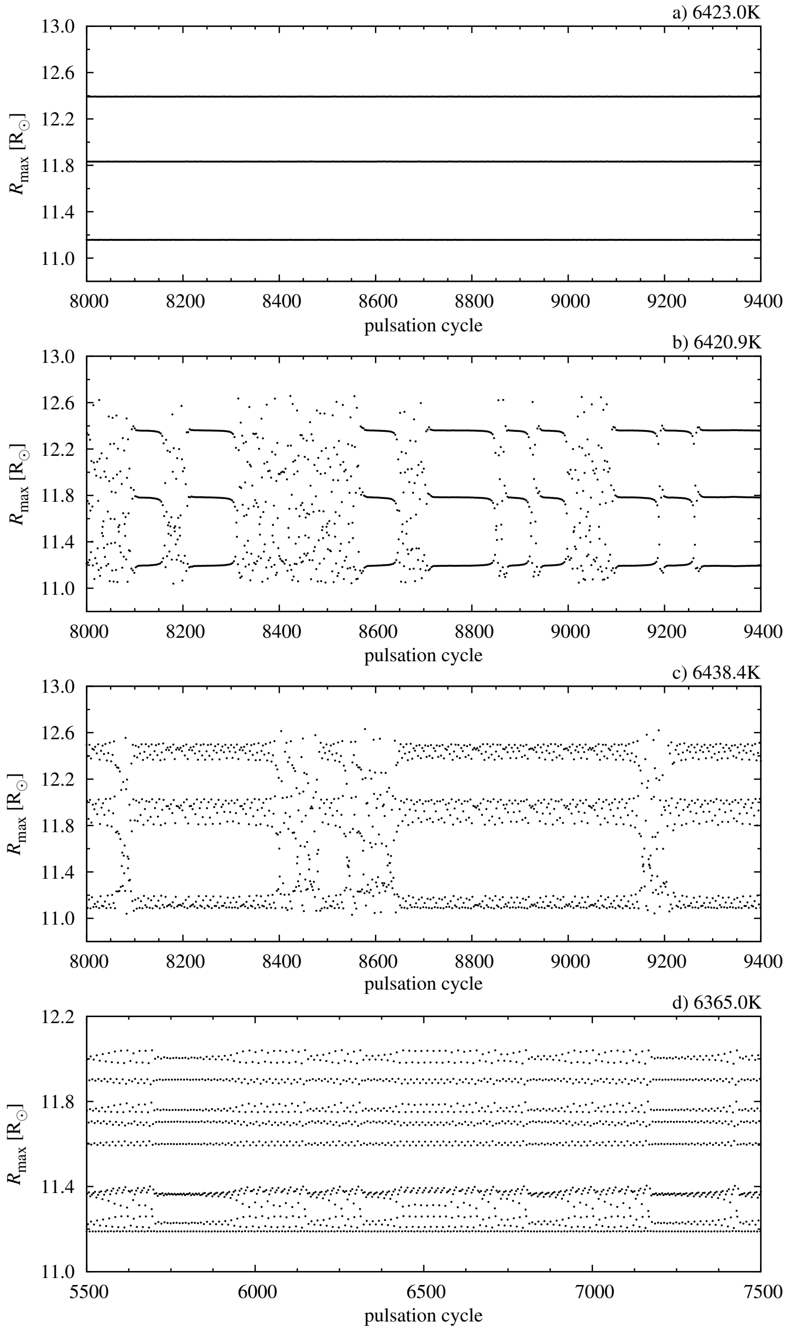

Fig. 2: Interesting dynamical behaviours we found in the models: a) period-3 cycle, b) type-I intermittency, c) crisis-induced intermittency, d) type III-intermittency.

Pulsation pattern repeats after three cycles of the fundamental mode. So far only period-2 (and the multiples in a period doubling cascade) were detected in the models. At the edges of period-3 window other interesting phenomena occur. As effective temperature is decreased we observe an intermittent route to chaos, which is a result of tangent bifurcation occurring at around 6421 K. The maximum radii variation in a slightly cooler model is demonstrated in panel b) of Fig. 2. We observe long phases of almost periodic, period-3 behaviour, interrupted by shorter burst of chaos. As effective temperature is decreased further, the almost periodic phases become shorter and chaotic bursts become longer. This is type-I intermittency (Pomeau & Manneville, 1980). At the hot edge of the period-3 window, around 6438 K, we observe an interior crisis bifurcation (Grebogi et al., 1982). The three chaotic bands formed in a series of period doubling bifurcations collide with the unstable period-3 cycle created in the tangent bifurcation at the cool side of the period-3 window. They merge into one large chaotic band (see Fig. 1), however we still observe a crisis induced intermittency depicted in panel c) of Fig. 2. The system evolves mainly in a phase-space defined by the former three-band chaotic attractor and only sporadically is scattered over a larger phase-space.

The last interesting phenomena, demonstrated in panel d) of Fig. 2, is type-III intermittency. It results from collision between stable periodic cycle and unstable period-doubled cycle born in sub-critical period doubling bifurcation (Pomeau & Manneville, 1980). In the discussed case the model switches intermittently between period-9 and period-18 cycles. It is well visible that the sub-harmonic (period-18) cycle grows up to some limiting amplitude and then switches back to cycle-9.

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