# Loops of Jupiter

Antoni Opolski[1]

1. Astronomical Institute, University of Wrocław
Kopernika 11, 51-622 Wrocław, Poland

Professor Antoni Opolski was actively interested in astronomy after his retirement in 1983. He especially liked to study the works of the famous astronomer Copernicus getting inspiration for his own work. Opolski started his work on planetary loops in 2011 continuing it to the end of 2012 . During this period calculations, drawings, tables, and basic descriptions of all the planets of the Solar System were created with the use of a piece of paper and a pencil only. In 2011 Antoni Opolski asked us to help him in editing the manuscript and preparing it for publication. We have been honored having the opportunity to work on articles on planetary loops with Antoni Opolski in his house for several months. In the middle of 2012 the detailed material on Jupiter was ready. However, professor Opolski improved the article by smoothing the text and preparing new, better drawings. Finally the article "Loops of Jupiter", written by the 99- year old astronomer, was published in the year of his 100th birthday.

## 1 Introduction

Let's imagine a young amateur astronomer who received a sky map. While watching the map he gets surprised by the fact that there are no orbits of planets marked there. Finally he finds out that planets appear in the sky regularly and periodically but each time in a different place. Therefore, for each planet only a zone in the sky, in which the planet appears, could be marked on the map. Besides, when we observe a motion of a planet in the sky we do not see the real movement of the planet but only the apparent one.

An issue of apparent orbits is usually omitted, or mentioned incidentally in astronomy textbooks and popular literature. Planets move usually along the ecliptic from west to east. This movement is called the direct motion. One of the coordinates defining the position of a planet is called the ecliptic longitude $\lambda$. This can be defined as an angle between the direction to a selected star and the direction to a planet. In the case of the direct motion an ecliptic longitude of a planet increases with time. Sometimes a planet moves seemingly in the opposite direction (retrograde motion) and then its ecliptic longitude decreases with time. The second coordinate of a planet is called latitude $\beta$ but we will not use it as it changes slightly. An observer on Earth determines the position of a planet in the sky relative to neighboring stars. Earth revolves around the Sun in an orbit close to a circle with a radius equal to the Astronomical Unit, $r_Z = 1$ AU. The change of the Earth's position relative to the Sun within six months equals 2 AU. The planets orbiting the Sun farther than Earth (superior planets) change their positions relative to Earth within a dozen of AU or so. The nearest stars are at distances of the order of $5 \times 10^5$ AU. Earth and the planets move and shift their positions, however the shifts are too small to change the directions to the neighboring stars. Therefore we assume that the directions of

Earth and the planets to the neighboring stars are constant regardless the fact that all the planets change their positions due to their orbital motions. For this reason in this article directions to selected stars are shown in the drawings as parallel lines, which looks as if stars moved along with the observer, but an error caused by this assumption is smaller than an annual parallax of a star, ie. is smaller than 1".

## 2 How the loops form

The formation of an apparent loop caused by the retrograde movement of a superior planet is shown in Fig. 1. The positions of Earth on its orbit around the Sun in

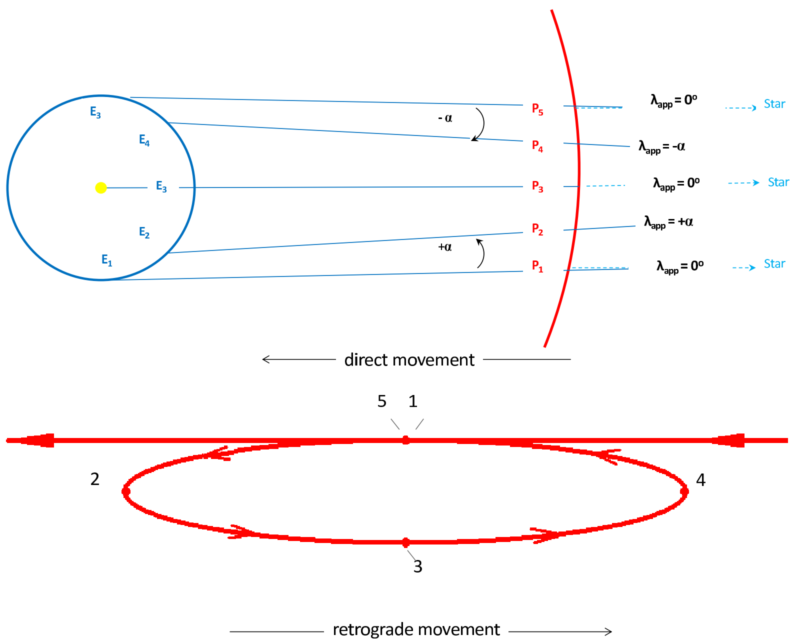


Fig. 1: **Top**: Creation of a planetary loop. $E_1$ - $E_5$: the positions of Earth on its orbit around the Sun in five characteristic moments. $P_1$ - $P_5$: the positions of a superior planet on its orbit around the Sun in the corresponding moments. **Bottom**: a scheme of an apparent planetary loop as seen from Earth.

five moments are marked with $E_1 - E_5$. The constant increase of both, an ecliptic longitude of Earth, $\lambda_E$, and an ecliptic longitude of a planet, $\lambda_p$, as seen from the Sun, is visible. When we combine the simultaneous positions (the same numbers) of Earth and the planet, and compare those directions with the direction from Earth to the reference star, it turns out that the apparent length of the planet, $\lambda_{app}$, sometimes decreases indicating the retrograde motion of the planet. From the position $P_2$, through $P_3$ to $P_4$, $\lambda_{app}$ of the planet decreases from $+\alpha°$, through $0°$ to $-\alpha°$. This results from the fact that the faster Earth overtakes the slower planet that seems to move backward. The same numbers indicate the corresponding points of the loop in Fig. 1b.

1. The beginning of the loop - the planet exhibits the direct movement.

2. The direct movement stops, the planet is stationary and next the retrograde movement starts.
3. The retrograde motion continues. Earth is in a straight line between the Sun and the planet (the planet is in opposition to the Sun). The distance between the planet and Earth is the smallest and equal the difference of the planet's and the Earth's radii.
4. The retrograde movement stops. The planet becomes stationary again and after that the direct movement starts.
5. The end of the loop.

The loop caused by the retrograde motion is only seen by an observer related to Earth exhibiting the orbital motion.

For an Earth's observer the orbital motion of our planet is imperceptible. Therefore the ancient astronomers recognized that this movement does not exist and Earth is immobile in the center of the world. As the result the geocentric concept held true for many centuries. Copernicus, who presented his heliocentric concept of the world, had no proof of the orbital movement of Earth. His appealing to the simplicity and the harmony was not convincing for everybody. The stellar aberration discovered by James Bradley in 1728 was the first unambiguous evidence of the existence of the Earth's orbital motion. The annual parallax of stars measured first in 1838 by Bessel was another phenomenon supporting the heliocentric system. The fact that a star copies the annual motion of Earth is the best proof of the existence of this motion. When the retrograde motion, the opposition and the loop will appear after some time again, Earth and the planet will be in different positions in space and the loop will look a little different. Such apparent, variable phenomena cannot be marked on sky maps.

## 3 Jupiter's example

We will now describe the Jupiter's example of a superior planet movement seen from Earth. Table 1 displays the necessary data.

Table 1: Orbital parameters

| | Earth | Jupiter |
|---|---|---|
| semi-major axis [AU] | $r_E = 1,0000$ | $r_J = 5,2028$ |
| sidereal period [days] | $T_E = 365,26$ | $T_J = 4333,29$ |
| heliocentric angular speed [°/d] | $u_E = 0,9865$ | $u_J = 0,08304$ |

We use a simplified model of the heliocentric system, where planets revolve around the Sun in the same plane along circular orbits of the same center where Sun is located. Each planet's movement is described by an appropriate stellar period, $T_p$. This is a time interval during which a planet revolves whole circle around the Sun. An angular velocity of this movement is constant and equals $u_p = 360°/T_p$.

The heliocentric coordinate system is built in the following way (see Fig. 2). The center is at the Sun. As an x-axis we take the line running through the Sun, Earth,

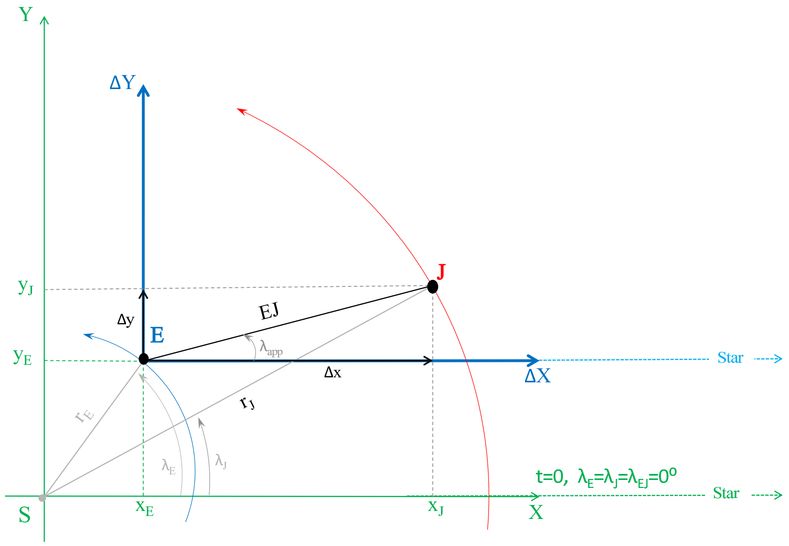


Fig. 2: The Earth's and Jupiter's coordinates in the heliocentric system (green axes) and the Jupiter's coordinated in the Earth-centered system (blue axes). S stands for the Sun, E - Earth, and J - Jupiter.

and Jupiter at opposition. Y-axis is just perpendicular. The moment of the opposition defines the beginning of a time-scale, t = 0d. The planets move along their orbits with appropriate angular velocities: $u_E$ for Earth and $u_J$ for Jupiter. The changes of their heliocentric ecliptic longitudes with time can be calculated as follows: $\lambda_E = u_E t$ for Earth and $\lambda_J = u_J t$ for Jupiter. Heliocentric longitudes and radii of planetary orbits describe planets' positions in the polar coordinate system. They transfer into the Cartesian coordinates as follows:

$$x_E = r_E \cos \lambda_E = r_E \cos u_E t, \quad y_E = r_E \sin \lambda_E = r_E \sin u_E t \tag{1}$$

$$x_J = r_J \cos \lambda_J = r_J \cos u_J t, \quad y_J = r_J \sin \lambda_J = r_J \sin u_J t \tag{2}$$

where $x_E$, $y_E$ mark the location of Earth and $x_J$, $y_J$ the location of Jupiter.

To describe the Jupiter's movements relative to Earth let me apply the Earth-centered system, with the axes ΔX and ΔY parallel to the axes of the heliocentric system. The center of this coordinate system moves together with Earth.

In the Earth-centered system the Jupiter's Cartesian coordinated are expressed as follows:

$$\Delta x = x_J - x_E = r_J \cos u_J t - r_E \cos u_E t \tag{3}$$

$$\Delta y = y_J - y_E = r_J \sin u_J t - r_E \sin u_E t \tag{4}$$

They can be transposed into polar coordinates (see Fig. 2) according to these expressions:

$$\Delta x = EJ \cos \lambda_{app}$$

$$\Delta y = EJ \sin \lambda_{app}$$

$$\tan \lambda_{\rm app} = \Delta x / \Delta y \quad (5)$$

$$(EJ)^2 = \Delta x^2 + \Delta y^2 \quad (6)$$

where $\lambda_{\rm app}$ is the apparent position of Jupiter seen from Earth and EJ is the distance between both planets.

Equations (1) to (6) allow to calculate the movements of Earth and Jupiter around the Sun ($\lambda_{\rm E}, \lambda_{\rm J}$), the distance between the planets (EJ), and the apparent movement of Jupiter in the Earth's sky ($\lambda_{\rm app}$). The parameters for some characteristic moments are listed in Table 2.

Table 2: Ecliptic longitudes of Earth $\lambda_E$ and Jupiter $\lambda_J$ in the heliocentric system, and the apparent positions of Jupiter in the Earth-centred system, $\lambda_{app}$, for the selected moments.

| position | marked in Fig. 1 | $t_d$ [d] | coordinates heliocentric $\lambda_E$[°] | $\lambda_J$[°] | geocentric $\lambda_{app}$[°] | EJ [AU] |
|---|---|---|---|---|---|---|
| $\lambda_{app} = 0°$ before opp. | 1 | -118.7 | -117.1 | -9.9 | 0.0 | 5.58 |
| stationarity before opp. | 2 | -60.3 | -59.5 | -5.0 | 5.0 | 4.69 |
| first opposition | 3 | 0.0 | 0.0 | 0.0 | 0.0 | 4.20 |
| stationarity after opp. | 4 | 60.3 | 59.5 | 5.0 | -5.0 | 4.69 |
| $\lambda_{app} = 0°$ after opp. | 5 | 118.7 | 117.1 | 9.9 | 0.0 | 5.58 |
| conjuction | C | 199.3 | 196.6 | 16.6 | 16.6 | 6.20 |
| $\lambda_{app} = 0°$ before 2nd opp. | 1 | 280.2 | 276.4 | 23.3 | 33.2 | 5.58 |
| stationarity before 2nd opp. | 2 | 338.7 | 334.1 | 28.1 | 38.1 | 4.69 |
| 2nd opposition | 3 | 398.9 | 33.5 | 33.1 | 33.0 | 4.20 |
| stationarity after 2nd opp. | 4 | 459.2 | 93.0 | 38.1 | 28.1 | 4.70 |
| $\lambda_{app} = 0°$ after 2nd opp. | 5 | 517.9 | 130.9 | 43.0 | 33.2 | 5.59 |

The specific configurations of the three objects, Earth, Jupiter and the Sun, repeat periodically. This period is called the synodic period. Figure 3 shows how the opposition of Jupiter repeats.

At the beginning Earth is located at $E_A$ while Jupiter is at opposition. After the synodic period, $T_{\rm syn}$, the opposition reappears, Earth locates between Jupiter and the Sun again, but this time at the point $E_C$. Earth overtook Jupiter by exactly one rotation. Within this period the Earth's heliocentric longitude rose by $T_{\rm syn} u_E$, and the Jupiter's longitude rose by $T_{\rm syn} u_J$, what is exactly 360° less:

$$T_{\rm syn} u_E = T_{\rm syn} u_J + 360°$$

This gives us the estimation of the synodic period:

$$T_{\rm syn} = 360° / (u_E - u_J) = 398.9^d = 1 \text{year} + 33.6^d$$

During $T_{\rm syn}$ the increase of the Earth's longitude is

$$\Delta\lambda_E = T_{\rm syn} u_E = 392.2° = 360° + 33.2°$$

and the Jupiter's is

$$\Delta\lambda_J = T_{\rm syn} u_J = 33.2°$$

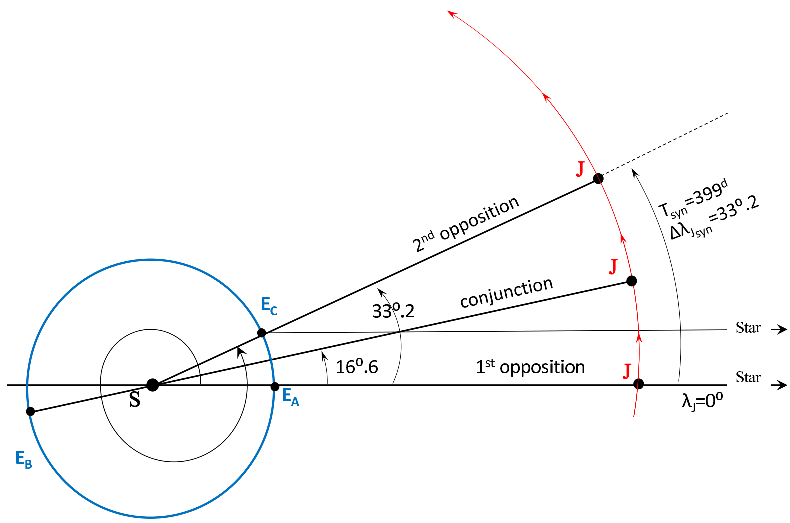


Fig. 3: The Earth's and Jupiter's movements between two consecutive oppositions, i.e. during one synodic period $T_{syn} = 399^d$.

At the half of the synodic period the is another linear configuration called conjunction. At this moment the Jupiter's longitude rose by $33.2° = 16.6°$, and the Earth's longitude by $16.6° + 180° = 196.6°$ The changes of the apparent longitude of Jupiter, $\lambda_{\rm app}$, are shown in Figure 4. Both forward and backward Jupiter's movements are seen. The five moments shown in Figure 1 together with the conjunction (K), when Jupiter is located behind the Sun, are also marked here.

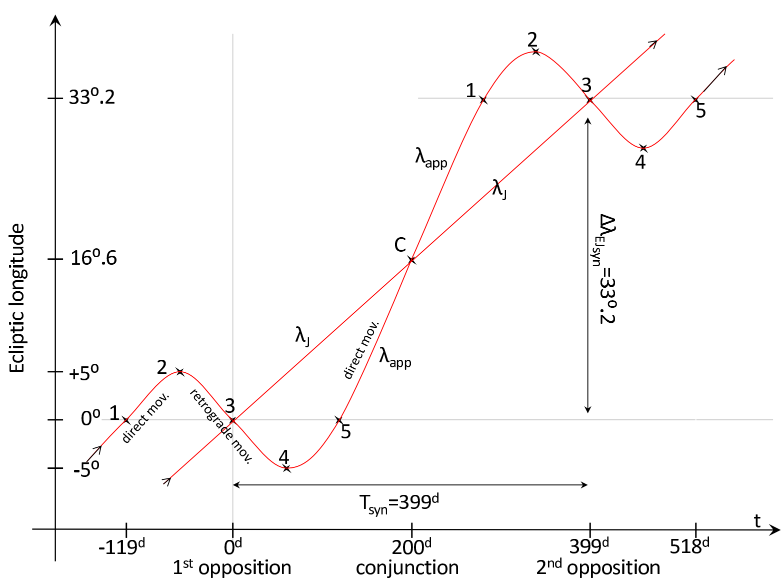


Fig. 4: Changes of the Jupiter's apparent $\lambda_{app}$ and heliocentric $\lambda_J$ longitudes in time.

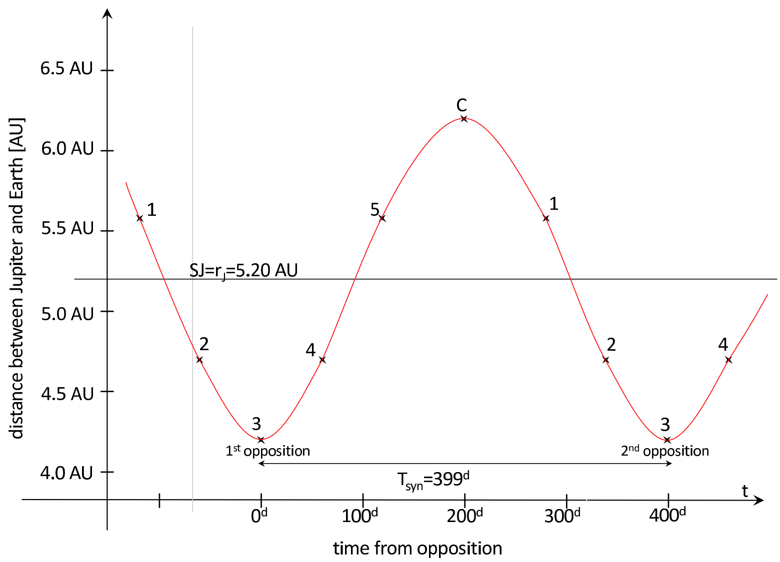


Fig. 5: Changes of the distance between Jupiter and Earth in time.

The straight line in Figure 4 represents the changes of the Jupiter's heliocentric longitude, which rises with the constant angular velocity $u_J = 0.083°/\text{day}$. Figure 5 presents the changes of the distance between the planets. One can see two minima corresponding to two consecutive oppositions. The intermediate maximum corresponds to the conjunction. Other points know form Figures 1 and Figure 4 are also marked. The horizontal line represents the distance between Jupiter and the Sun.

Let's now see how the Jupiter's movement complicates when we assume still Earth. Figure 6 shows the loop that passes through the points J1 - J5. Its length J1 – J3 equals 1.38 AU and width J2 – J4 equals 0.92 AU. The circle now represents the Sun's movement around the still Earth. The distance between the Sun and Jupiter is constant and equal to $r_J$. Figure 6 presents also how the Jupiter's loop appears on the sky. The distance between Earth and Jupiter changes from 4.20 AU during an opposition to 6.20 AU during a conjunction. At the next opposition Jupiter's apparent longitude rises by $\Delta\lambda_{\text{app}} = 33°12'$. The angular span of the backward movement for Jupiter is 10°. Although Jupiter's movement in Earth-centered system is more complicated, its average value is exactly the same as in the heliocentric system:

$$\bar{u}_J = \frac{\Delta\lambda_{\text{app}}}{T_{\text{syn}}} = \frac{33°12'}{398,9^{\text{d}}} = 0.083°/\text{d} = u_J$$

The ratio of the orbital and the synodic periods for Jupiter is equal to the ratio of two increments of longitude:

$$\frac{T_J}{T_{\text{syn}}} = \frac{\Delta\lambda_{SJ}}{\Delta\lambda_{\text{app}}} = \frac{360°}{33°12'} = 10.9$$

This shows that while in the heliocentric system Jupiter revolves around the Sun along simple circular orbit, in the geocentric system it has to dance quite a complicated loop almost eleven times. Possibly this was the inspiration for Copernicus when he

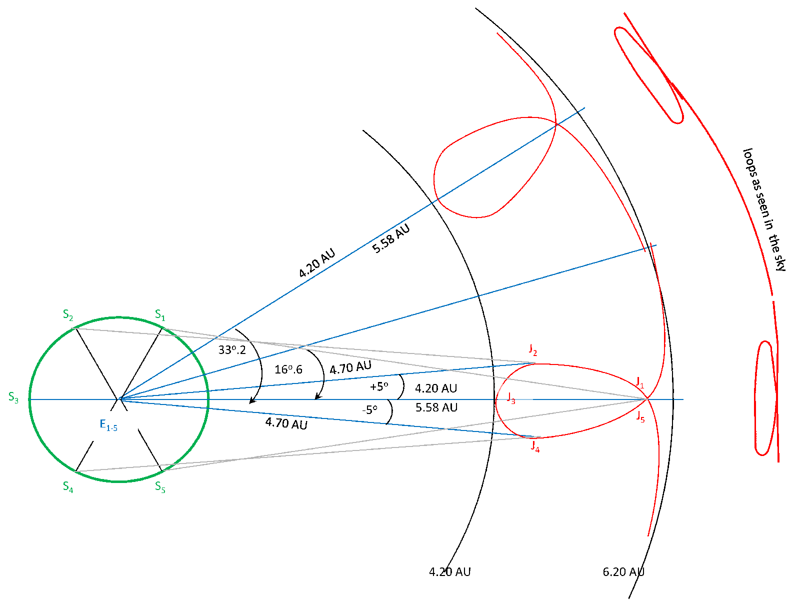


Fig. 6: The Jupiter's movement in the Earth-centered system. $S_1$ - $S_5$ are the Sun's positions corresponding to the Earth's positions $E_1$ - $E_5$. The corresponding positions of Jupiter are $J_1$ - $J_5$.

presented the simplicity and harmony of the heliocentric system in comparison with complicated planetary movements in the geocentric system.