Relative Clustering Ratios in Modified Gravity Cosmologies

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We make use of large state-of-the-art N-body simulations to measure and model the two-point statistics of a variety of models, which include $f(R)$ gravity and the normal branch of the Dvali-Gabadadze-Porrati (nDGP) braneworld. We find that the statistical significance of these signals largely diminishes due to a strong degeneracy between MG-enhanced clustering and modified tracer bias; therefore, we consider the relative clustering ratios as a less bias-dependent statistic. The clustering ratios foster the differences between modified gravity and GR models, and such departures could be measured for the linear distortion parameter if non-linear effects at intermediate scales are correctly modeled. We also find that the selection of an optimal tracer sample depends on a particular statistic and gravity model to be considered. Finally, our results indicate that the clustering ratios give great promise to search for signatures of MG in the large-scale structure.

1 Introduction

The observational data, together with a phenomenological description of the fundamental building blocks of the Universe, have led to the current standard cosmological model Lambda-Cold Dark Matter, ΛCDM (see e.g. Hamana et al., 2015; Alam et al., 2017; Abbott et al., 2018; Pacaud et al., 2018; Aghanim et al., 2020). In an attempt to accurately describe the observations on cosmological scales, the ΛCDM model is based on the theory of general relativity (GR) and lies on the assumption that the Universe is dominated by dark energy (DE) in the form of a cosmological constant (responsible for the accelerated expansion), and a cold dark matter (CDM) component that drives structure formation. Both the unknown nature of DE and CDM and the lack of GR tests on cosmological scales have raised many questions without a satisfactory physical explanation neither from a theoretical nor observational point of view (Albrecht et al., 2006; Frieman et al., 2008). The fissures in our understanding of the Universe have motivated the interest in cosmological models beyond GR, among them, the models based on $f(R)$ gravity and braneworld are notable because of their generality and rich phenomenology (De Felice & Tsujikawa, 2010; Sotiriou & Faraoni, 2010). Modified gravity (MG) cosmologies appear to be one of the most promising explanations for cosmic acceleration (Joyce et al., 2016), since they can satisfy simultaneously solar system constraints and be consistent with the cosmological background of the ΛCDM model (Uzan, 2011; Will, 2014; Pezzotta et al., 2017; Collett et al., 2018). Among the proposed modifications of GR, we consider both the chameleon $f(R)$ modified gravity and the normal branch Dvali-Gabadadze-Porrati (nDGP) brane world model (Dvali et al., 2000) as well as the fiducial ΛCDM scenario. The effects of modifications of gravity are naturally imprinted in the distribution of large-scale structures, and clustering analysis stands out as an important probe of the underlying cosmological model. The two-point correlation function (henceforth 2PCF) allows us to describe the clustering of cosmic tracers. Here a specific feature suitable to test gravity models is redshift-space distortions (RSD), which induce anisotropy in the observationally derived two-point statistics of the cosmic tracers.

In this work we focus on the two-point statistics in ΛCDM and selected MG models by exploiting large state-of-the-art N-body simulations. We mostly deal with clustering information in the redshift space, in which observations are performed, in order to connect with the observational expectations of current galaxy surveys. We investigate one of the main systematics that plagues the derivations of cosmological parameters from redshift surveys, namely the halo and galaxy bias, and its relation with the growth rate of matter perturbations, which describes the evolution of overderdensities in the matter density field and that can be used to search for imprints of alternative MG models.

2 Hu-Sawicki $f(R)$ and $nDGP$ cosmologies

2.1 Hu-Sawicki $f(R)$ gravity

The $f(R)$ gravity model is a natural extension of GR that can be obtained by modifying the Einstein-Hilbert action by adding a scalar function, $f(R)$, to the Ricci scalar, R. In this framework, the cosmic acceleration of the Universe is explained by a $f(R)$ function, without the need for any form of cosmological constant. The additional gravitational force – often referred to as the "fifth force" – is highly environment-dependent and its screening is obtained via the so-called "chameleon mechanism". Hu & Sawicki (2007) proposed the following $f(R)$ function that satisfies the solar system constraints and mimics background solutions of the ΛCDM model:

$$f(R) = -m^2 \frac{c_1 \left(\frac{R}{m^2}\right)^n}{c_2 \left(\frac{R}{m^2}\right)^n + 1}, \tag{1}$$

where the mass scale m is defined as $m^2 \equiv H_0^2 \Omega_{\rm M}$, and c_1, c_2 and n are non-negative free parameters of the model (Hu & Sawicki, 2007). Choosing $c_1/c_2 = 6\Omega_\Lambda/\Omega_{\rm M}$, where Ω_Λ and $\Omega_{\rm M}$ are the dimensionless density parameters for vacuum and matter, respectively, the $f(R)$ model reproduces the background expansion history of the concordance model. The extra degree of freedom can be expressed in terms of a scalaron field $f_{\rm R} \equiv df(R)/dR$. In particular, for the condition $c_2(R/m^2)^n \gg 1$ and $n = 1$, the Hu-Sawicki $f(R)$ model is completely determined by one single free parameter which in turn can be rewritten in terms of the dimensionless scalar at the current epoch, $f_{\rm R0}$.

2.2 nDGP gravity

The normal branch of the Dvali-Gabadadze-Porrati (nDGP) model is a natural extension of the DGP model (Dvali et al., 2000; Koyama, 2007). This model has its origins in string theory and considers a 4+1-dimensional Minkowski space as the DGP model does. Nevertheless, the nDGP model introduces a free parameter that characterizes the scale at which the four-dimensional gravity propagates through the extra dimensions. This scale, named crossover scale $r_{\rm c}$, is obtained as half the

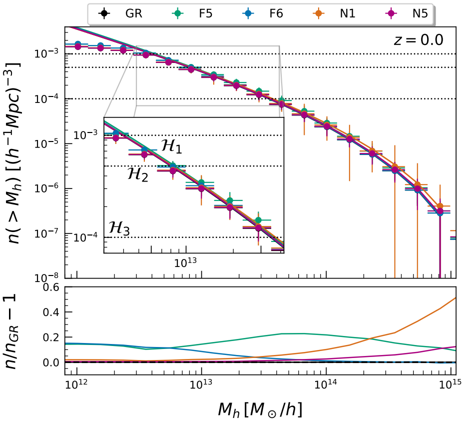

Fig. 1: Cumulative halo mass function for the different ELEPHANT MG models taken at redshift $z = 0$.

DM particles	1024^3
Particle mass	$7.8 \times 10^{10} M_\odot h^{-1}$
Box side	$1024 h^{-1}$ Mpc
Ω_m	0.281
Ω_b	0.046
Ω_ν	0.
Ω_r	0.
h	0.697
A_s	2.41e-09
n_s	0.971
σ_8	0.820

Fig. 2: Short description of the simulation boxes and cosmological parameters used in the ELEPHANT runs that are consistent with the WMAP9 best-fit values (Hinshaw et al., 2013).

ratio of the five-dimensional Newton's constant $G^{(5)}$ to the universal gravitational constant G. The nDGP model is fully described by the parameter Ω_rc, which is defined in terms of the cross-over scale as $\Omega_\mathrm{rc} \equiv 1/(2r_\mathrm{c}H_0)^2$, with H_0 denoting the present-day value of the Hubble parameter, and the above quantities are expressed assuming $c = 1$. The magnitude of $H_0 r_\mathrm{c}$ quantifies the order of departure from GR. We consider two nDGP variants with its free parameter $H_0 r_\mathrm{c}$ taken to be $r_\mathrm{c}H_0 = 5$ and 1, respectively labeled as N5 and N1, and that represent weak and medium deviation from the ΛCDM model. Due to the enhancement of the Newtonian gravity that occurs mainly on large scales, the nDGP model exhibits a constant enhancement in the growth rate of structure in the linear regime, while at smaller scales (nonlinear regime), the gravity enhancement is effectively suppressed by Vainshtein screening (Vainshtein, 1972; Li et al., 2013).

3 N-body simulations and redshift-space distortions

We consider a subset of the ELEPHANT simulations (Extended LEnsing PHysics using ANalaytic ray Tracing; introduced by Cautun et al. 2018), which consist of a suite of dark-matter-only N-body simulations of ΛCDM, and of MG theories, including the $f(R)$ models (F5 and F6), and nDGP (N1 and N5). They have five independent realisations of each gravity model in the redshift range $z \in \{0, 0.3, 0.5\}$. The simulations have been run using the `ECOSMOG` code (Li et al., 2012), a modified version of the publicly available N-body and hydrodynamical simulation code `RAMSES` (Teyssier, 2002) and dark matter (DM) halos were identified using the `ROCKSTAR` halo finder. Table 2 shows a summary of the features of the ELEPHANT simulations including the cosmological parameters which are consistent with those from the WMAP9 collaboration (Hinshaw et al., 2013).

Figure 1 shows the cumulative mass function of DM halos in each MG model. This quantity describes the number density of halos as a function of their mass above a defined threshold. Our samples are defined according to the SDSS main

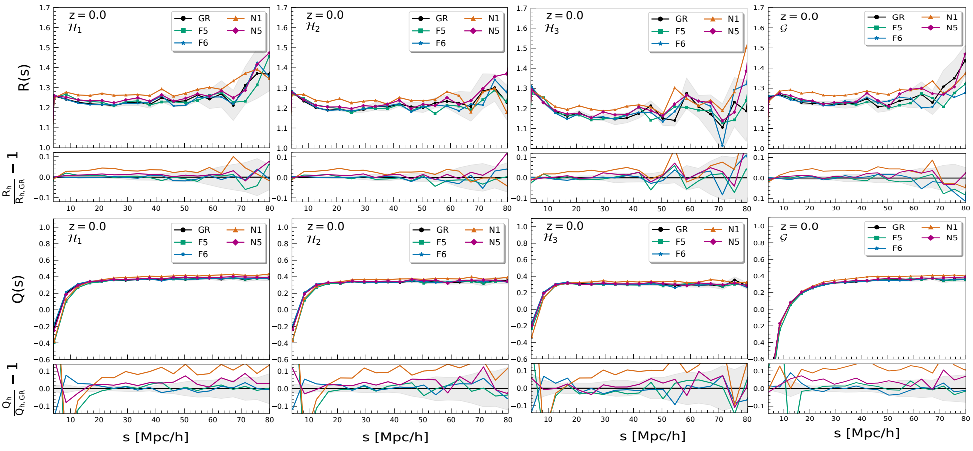

Fig. 3: The multipole moment ratios: $R(s)$ and $Q(s)$ at $z = 0$ for the different halo (galaxy) samples of the ELEPHANT MG models as indicated by the labels. The lower panels show the percent difference of MG models with respect to the ΛCDM prediction.

galaxy sample density. The dashed horizontal lines plotted in Fig. 1, represent the numerical densities of each sample being: $\mathcal{H}_1 = 10^{-3}h^3\,\mathrm{Mpc}^{-3}$, $\mathcal{H}_2 = 5\times 10^{-4}h^3\,\mathrm{Mpc}^{-3}$, $\mathcal{H}_3 = 10^{-4}h^3\,\mathrm{Mpc}^{-3}$. Besides analyzing DM halos, we also consider a galaxy sample built from the ELEPHANT suite using a halo occupation distribution (HOD) model. The galaxy sample follows closely the mass function of the sample $\mathcal{H}_2$. The bottom panel of Fig. 1 shows the relative deviation with respect to the ΛCDM model.

We constructed redshift-space mock catalogues using the distant observer approximation for each halo and galaxy sample of the ELEPHANT suite. The overall impact of the peculiar velocity of cosmic tracers generates a distortion effect, that makes the spatial distribution of tracers appear squashed and distorted in the redshift space. Two types of distortions can be distinguished depending on the magnitude of the peculiar velocity with respect to the Hubble flow velocity (Hamilton, 1998): 1) the *Fingers of God* (FoG) effect (Jackson, 1972), that is prevalent in the non-linear regime of the structure formation when the tracer velocities may exceed the Hubble flow, and 2) the *Kaiser effect* (Kaiser, 1987), in which the density contrast appears squashed due to the coherent motions of the tracers as they fall towards virialized structures.

4 Relative clustering analysis

The degree of anisotropy present in the clustering due to RSD can be better understood by making explicit the azimuthal symmetry of the 2D 2PCF (Hamilton, 1998). Therefore, it can be expressed as a Legendre expansion such that $\xi(s,\mu) = \sum_l \xi_l(s)\mathcal{L}_l(\mu)$, where s is the separation vector of the tracers, $\mathcal{L}_l$ are the Legendre polynomials of degree l and $\mu \equiv \cos\theta$ is the directional cosine between the line-of-sight direction. The two first non-null multipole moments of the 2PCF are the monopole, ξ_0, and quadrupole, ξ_2. They represent the angular averaged 2PCF and the leading anisotropies due to RSD, respectively. Combining both the multipole moments of the 2PCF and the linear perturbation theory for the density field, it is possible to obtain two expressions to constrain the growth of cosmic structures written as follows (Hamilton, 1998):

$$R(s) = \frac{\xi_0(s)}{\xi_0(r)} = 1 + \frac{2\beta}{3} + \frac{\beta^2}{5}, \tag{2}$$

$$Q(s) = \frac{\xi_2(s)}{\xi_0(s) - \frac{3}{s^3}\int_0^s ds'\xi_0(s')s'^2} = \frac{\frac{4}{3}\beta + \frac{4}{7}\beta^2}{1 + \frac{2\beta}{3} + \frac{\beta^2}{5}}. \tag{3}$$

Here, $\beta \equiv f/b$ is the linear distortion parameter, f the linear growth rate and b the linear bias parameter. Using the Landy & Szalay (1993) estimator we compute the 2D 2PCF[1] in both real- and redshift-space for each of the ELEPHANT samples ($\mathcal{H}_1$, $\mathcal{H}_2$, $\mathcal{H}_3$ and $\mathcal{G}$), which were selected in each MG model. Figure 3 shows the multipole moment ratios $R(s)$ and $Q(s)$ at $z = 0$ for the different ELEPHANT MG models and three different number densities of the halo population as well as the galaxy sample. Although the quadrupole ratio $Q(s)$ may be more susceptible to noise than only the $R(s)$ ratio, the main advantage of the former is that it does not require prior knowledge about the underlying shape of the real-space clustering unlike $R(s)$. Once $R(s)$ or $Q(s)$ are determined, it is possible to obtain an estimate of the β parameter, however it has been shown that these statistics only perform well in the linear regime. These estimators allow us to quantify the deviations in clustering between GR and MG following linear theory prescription as shown in Fig. 3.

The sample density can introduce a strong degeneracy with the MG effects encoded in the β parameter. To eliminate these effects in the linear order, which contribute most, we extend the so-called *clustering ratios* (Arnalte-Mur et al., 2017) to the multipole moments of the 2PCF. In this new representation, the *clustering ratios* are defined as follows

$$\mathcal{R}_l\left(s, \mathcal{H} \mid \mathcal{H}_{\text{ref}}, s_{\text{ref}}\right) = \frac{s^2\xi_l(s \mid \mathcal{H})}{s_{\text{ref}}^2\, \xi_l\left(s_{\text{ref}} \mid \mathcal{H}_{\text{ref}}\right)}, \tag{4}$$

where $\mathcal{H}_{\text{ref}}$ is a reference halo (galaxy) sample and s_{ref} is a fixed reference comoving scale. The simplest criterion for selecting the halo reference sample, $\mathcal{H}_{\text{ref}}$, is to take the one with less noise in the estimated linear bias. That will increase the signal to noise ratio of the remaining samples and also allow us to better distinguish the effects of MG from GR. In this analysis we consider the densest halo population as a reference sample, i.e,. $\mathcal{H}_{\text{ref}} = \mathcal{H}_1$, whose number density is $\bar{n} = 10^{-3}h^3\text{Mpc}^{-3}$. Figure 4 displays the redshift-space relative clustering ratios, monopole and quadrupole, at the reference scale of $64h^{-1}\,\text{Mpc}$. We find that the MG models which exhibit most deviations in this statistic compared to ΛCDM are the nDGP family. In the monopole clustering ratio, $\mathcal{R}_0$, we see a significant amplitude suppression below the GR signal at all scales. In some cases also the $f(R)$ models show visible departures from GR, although the clustering ratio at this specific reference scale, $s_{\text{ref}} = 64h^{-1}\,\text{Mpc}$, is not effective to untangle the screening effects of modifications

[1]We use the publicly available code Correlation Utilities and Two-point Estimation (CUTE, Alonso, 2012).

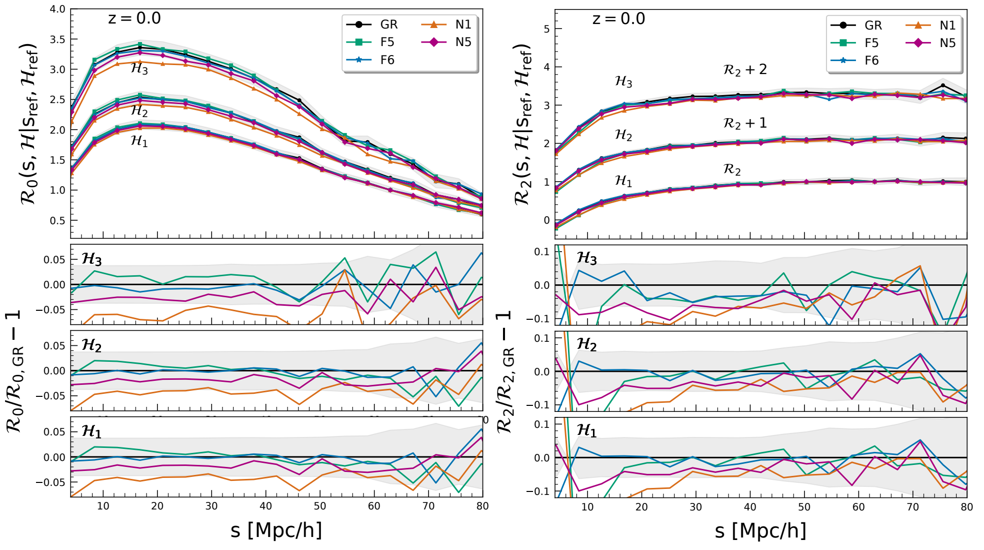

Fig. 4: The MG clustering ratios with respect to the ΛCDM measurements for both the monopole (*left*) and quadrupole (*right*) of the 2PCF of halos, as indicated by the labels, at the reference scale $s_{\rm ref} = 64h^{-1}\,{\rm Mpc}$. The gray-shaded areas correspond to the standard deviation for GR over 15 measurements of the 2PCF estimation.

of gravity up to 5%. As far as the $f(R)$ cosmologies are concerned, our findings are in agreement with Arnalte-Mur et al. (2017). In particular, we find that the clustering ratio of the MG models resembles closely the GR case. In fact, the deviation from GR in the monopole $\mathcal{R}_0$ can be better distinguished when considering different reference scales and this leads to interesting conclusions. For a detailed discussion of the clustering ratios of multipoles and clustering wedges in both linear and non-linear regime of the structure formation see García-Farieta et al. (2021).

5 Summary

We presented a systematic clustering analysis in redshift space of DM halos and galaxy catalogues of the $f(R)$ and nDGP cosmologies using state-of-the-art N-body simulations. We measured the two-point statistics and found that the significance of the clustering diminishes due to a degeneracy between MG-enhanced signal and tracer bias. Therefore, we extended the analysis introducing the relative clustering ratios of the multipole moments of the 2PCF, which promise to be less susceptible to growth rate-bias degeneracy. Based on the extension of this statistic it is therefore feasible to obtain model-independent constraints as long as the distortion parameter of the reference sample is optimally constrained at a given reference scale. Finally, our results indicate that the clustering ratios give great promise to search for signatures of MG in current and upcoming galaxy surveys.

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