Repulsion Between Two Super-Earths Due to Wave Planet Interactions

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We study the orbital evolution of a system of two super-Earths embedded in a protoplanetary disk close to a first order resonance. We focus on a repulsion mechanism connected to wave planet interactions. This works through angular momentum carried by waves emitted by one planet being transferred to the vicinity of the other planet and subsequently to it through horseshoe drag.

From hydrodynamical simulations, we found that the initial convergent migration of two super-Earths is converted to become divergent, when approaching first order resonances, thus preventing the system from becoming locked into them. We give approximate conditions for a planet to be effectively repelled due to the wave planet repulsion mechanism. These imply that the planet should be able to form a partial gap in the disk and there should be enough material in it to enable angular momentum to be efficiently transferred to the associated planet through horseshoe drag. The numerical simulations indicate that when these conditions for the wave planet repulsion mechanism are satisfied, divergent migration occurs thus preventing capture into resonance.

1 Introduction

The orbital evolution of the planets in multi-planetary systems takes place once they have formed in a gaseous protoplanetary disk. Analytical and numerical studies adopting simple migration models indicate that pairs of planets can be captured into mean-motion resonances (MMRs), with their period ratio extremely close or equal to the ratios of small integers, as a result of convergent migration. However, observations show that such resonance configurations are not commonly seen in multi-planetary systems. Moreover, the observed planet pairs which are near to MMRs tend to have period ratios slightly larger than those required for strict commensurability (Lissauer et al., 2011; Fabrycky et al., 2014; Steffen & Hwang, 2015). This feature has also been found in some low-mass planet pairs where both orbital periods exceed 10 days. Detailed information is given in Tab. 1.

Unlike some mechanisms that have been proposed to explain such deviations from strict commensurability after a migration phase in the protoplanetary disk, the one we focus on here can operate for planet pairs at large distances from the central star. Repulsion between the two planets occurs on account of the angular momentum transferred by density waves launched by one planet to the horseshoe region of the other planet and subsequently to that planet itself. This repulsion can prevent the planets converging to a strict commensurability.

This process has been investigated for a system consisting of an inner giant planet and an outer super-Earth using both global two-dimensional hydrodynamic calculations and local shearing box simulations (Podlewska-Gaca et al., 2012). The

Tab. 1: Two-planet systems with the period ratios near the first-order resonances

Planet pair	Period (days)	Mass (M_*)	MMR	Relative deviation	Reference
Kepler-59b Kepler-59c	11.8715 17.9742	1.10×10^{-5} 1.02×10^{-5}	3:2	0.0141	Saad-Olivera et al. (2020)
Kepler-128b Kepler-128c	15.090 22.804	2.12×10^{-6} 2.48×10^{-6}	3:2	0.0112	Hadden & Lithwick (2016)
Kepler-177b Kepler-177c	35.860 49.409	1.90×10^{-5} 4.79×10^{-5}	4:3	0.0445	Vissapragada et al. (2020)

results showed that the inward migration of the super-Earth could be stopped or even reversed due to the angular momentum transferred by the outward propagating density waves launched by the inner giant planet. This mechanism has also been studied for pairs of Saturn-like planets pairs and a system of two Uranus-like planets (Baruteau & Papaloizou, 2013). It was found that the divergent migration of the planets caused by the wave planet interactions was particularly efficient when at least one planet opens a partial gap in the disk.

In this work, we extend previous studies to consider a system of two super-Earths which are able to open partial gaps in the vicinity of their orbits using two-dimensional hydrodynamic simulations. The aim is to investigate the effectiveness of the wave planet repulsion mechanism in preventing the low-mass planets from being captured into resonances. Finally, we provide simple approximate conditions to indicate when the repulsion mechanism is able to account for the occurrence of divergent migration of two planets, that would undergo convergent migration when not interacting in the disk, and verify them using the hydrodynamic simulations. The full study can be found in a long version of this paper (Cui et al., 2021).

2 Disk model and numerical setup

We consider a system of two super-Earths orbiting around a central star in a protoplanetary disk. The masses of two planets are represented by the mass ratios $q_{\rm i} = m_{\rm i}/M_*$, where M_* and $m_{\rm i}$ respectively indicate the central stellar mass and the planetary mass with i=1 denoting the inner planet and i=2 denoting the outer one. A two-dimensional disk model is adopted. We adopt a cylindrical polar coordinate system (r, ϕ, z) with its origin located at the position of the central star. The disk gas obeys a locally isothermal equation of state which gives the vertically integrated pressure to be $P = \Sigma c_{\rm s}^2$, where $c_{\rm s}$ is the sound speed. This is related to the vertical scale height of the disk H through $H = c_{\rm s}/\Omega$ with Ω being the Keplerian angular velocity. The aspect ratio of the disk, defined as $h = H/r$, is assumed to be constant which leads to the radial temperature profile being proportional to r^{-1}. The disk self-gravity is neglected.

The system of units adopted in the simulations is as follows: The unit of mass is the central stellar mass M_*. The unit of length is the initial orbital radius of the inner planet r_1. The time unit is such that the initial orbital period of the inner planet $P_1 = 2\pi$.

In our simulations, both planets are initially located in circular orbits with zero eccentricity. The initial orbital radii for the inner and outer planets are $r_1 = 1$ and

$r_2 = 1.48$, respectively, which gives an initial period ratio larger than 3:2. In this work, we consider two initial surface density profiles $\Sigma(r)$. The first takes the form of a power law given by

$$\Sigma(r) = 8 \times 10^{-5} r^{-1/2}. \tag{1}$$

The second modifies a power law for $r < 1$ to introduce a central cavity. It is given by

$$\Sigma(r) = \begin{cases} 1.5 \times 10^{-5}(5r - 1) & \text{for} \quad r_{\min} < r < 1 \\ 6.0 \times 10^{-5} r^{-1/2} & \text{for} \quad 1 \leq r < r_{\max}, \end{cases} \tag{2}$$

where $r_{\min}$ and $r_{\max}$ are the inner and outer boundaries of the computational domain, respectively. In some simulations, the initial surface density was given by Eq. 2 to guarantee the convergent migration of two planets in the early stages of the evolution.

In this work, the hydrodynamical equations governing the disk evolution are solved by using the numerical code FARGO3D (Benitez-Llambay & Masset, 2016). Effects resulting from the turbulence are taken into account by adopting a constant kinematic viscosity ν. The computational domain in the radial direction extends from $r_{\min} = 0.2$ to $r_{\max} = 2.6$ and it covers the full 2π in azimuth. The uniformly spaced computational grid has 900 cells in radius and 1800 cells in azimuth. The reference frame rotates with the initial Keplerian angular velocity of the inner planet.

Standard outflow boundary conditions are applied with wave killing-zones (de Val-Borro et al., 2006) operating in the vicinity of both the inner and outer radial boundaries of the computational domain. In all simulations, the disk aspect ratio adopted was $h = 0.02$ and the viscosity was taken to be $\nu = 1.2 \times 10^{-6}$. The gravitational potential of the planets is smoothed by making use of a conventional softening parameter b equal to $0.6h$.

3 Divergent migration of two super-Earths in a protoplanetary disk

In this section, we describe the divergent migration of two super-Earths near the 3:2 MMR in a protoplanetary disk using 2D hydrodynamic simulations. Then we give conditions for the the wave planet repulsion mechanism to be effective in leading to this phenomenon, verifying them in systems with different planetary mass ratios.

3.1 Results of hydrodynamic simulations

We consider the simulation of a system of two super-Earths migrating in a protoplanetary disk in which the mass of the inner and outer planets are respectively taken to be 1.3×10^{-5} and 1.185×10^{-5} while the initial surface density profile is adopted as Eq. 1. The evolution of the period ratio, semi-major axes, eccentricities of two planets and the 3:2 resonance angles is illustrated in Fig. 1. We can see that the planets arrive near the 3:2 MMR after around 6000 orbits as a result of convergent migration. Meanwhile, both eccentricities are excited and resonance angles begin to librate. Subsequently, the migration rate of the outer planet slows down. The relative migration of planets changes from convergent to divergent with increasing period ratio. Thus the planets move away from the 3:2 MMR.

We illustrate the azimuthally averaged surface density profile and a contour plot of the disk surface density in the vicinity of the two planets at 8560 orbits when divergent migration occurs in the simulation in Fig. 2. It is clear that both planets develop a partial gap in the vicinity of their orbits. In addition, for the disk parameters adopted in the simulation, the partial gaps are well separated. Similar results to these are obtained when the initial surface density profile is taken to be given by Eq. 2 corresponding to the disk having a central cavity.

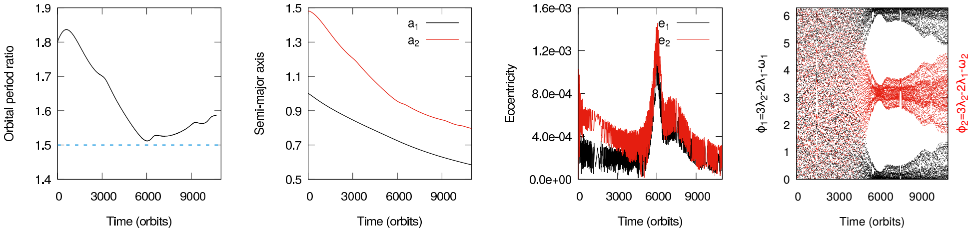

Fig. 1: Evolution of the orbital period ratio, semi-major axes, eccentricities and the 3:2 resonance angles for the simulation of two super-Earths with $q_1 = 1.3 \times 10^{-5}$ and $q_2 = 1.185 \times 10^{-5}$ migrating in a protoplanetary disk with the initial surface density given by Eq. 1. The horizontal dashed blue line in the leftmost panel indicates the location of the 3:2 commensurability. Here and in other Figures the time unit, or 'orbit', is the initial orbital period of the inner planet.

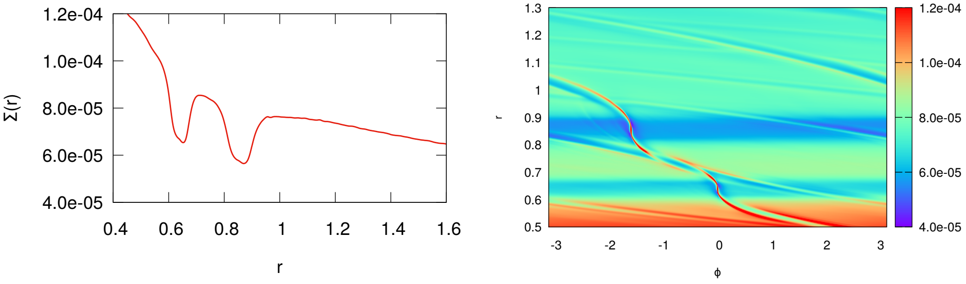

Fig. 2: The azimuthally averaged surface density profile (*left*) and a contour plot of the disk surface density in the vicinity of two planets at $t \sim 8560$ orbits (*right*) obtained from the simulation shown in Fig. 1.

We have compared the migration rate of the outer planet that would occur were it isolated with that found from the simulation with two planets. We found an additional torque acted on the outer planet, requiring a mechanism for transferring angular momentum between the planets. Since a similar phenomenon was found in simulations of two migrating planets without mutual gravitational interaction by Baruteau & Papaloizou (2013), we infer that the interaction between them is mediated by the presence of the disk. This can occur through wave transport. When that operates efficiently, convergent migration of two planets can be switched to divergent.

3.2 Effectiveness of the wave planet repulsion mechanism

We propose that wave planet interactions can be responsible for the repulsion between planets found in the simulation. This mechanism invokes propagating density waves excited by one planet dissipating in the co-orbital region of the other planet in the system. Subsequently the angular momentum that was carried by the waves is transferred to that planet through horseshoe drag. Therefore, an additional torque acts on it, affecting its migration in the disk. When such a torque acts effectively on one or both planets, their relative migration can be changed from convergent to divergent. We found approximate conditions for its effective action on a particular planet, which imply that, (1) the planet should be massive enough to open a partial gap, (2) there should be adequate material in its horseshoe region to enable the angular momentum carried by the density waves launched by the other planet to be transferred to it. Thus, if the mechanism works effectively on the outer planet, the first condition leads to the requirement that

$$\frac{q_2}{h^3} > 1. \tag{3}$$

The second condition expresses the requirement that the maximum possible torque due to horseshoe drag exceeds that produced by the wave input. This becomes a condition on the surface density in the gap taking the form

$$\frac{\Sigma_{\mathrm{min},2}}{\Sigma_{\mathrm{un},2}} > \frac{\pi q_1^2}{h^2 q_2}, \tag{4}$$

where $\Sigma_{\mathrm{min},2}$ and $\Sigma_{\mathrm{un},2}$ respectively represent the azimuthally averaged surface density in the partial gap produced by the outer planet and its unperturbed value.

On the other hand, we can consider the case when the inner planet receives the angular momentum transferred through density waves generated by the outer planet. In that case the conditions are obtained by interchanging q_1 and q_2 and the subscripts 1 and 2 attached to the surface densities in Eq. 3 and Eq. 4.

In order to verify the conditions mentioned above, we extended the study presented in Fig. 1 to consider planet pairs with a range of mass ratios while adopting the initial disk surface density profile given by Eq. 2. Here we consider four planet pairs in which the outer planet's mass is fixed to be 1.185×10^{-5} but the inner planet's mass is taken to be 1.185×10^{-5}, 1.3×10^{-5}, 1.95×10^{-5} and 2.6×10^{-5}. From Eq. 3 and Eq. 4 we find that the conditions for the effectiveness of the wave planet repulsion mechanism for both planets are satisfied in all of these cases. Thus we anticipate the possibility of divergent migration.

The results of the simulations with different masses of the inner planet, q_1, are illustrated in Fig. 3. We can see that for the case in which the inner planet has the lowest mass indicated by the orange line, the planets can reach the 3:2 MMR but cannot remain there. For the cases with higher values of q_1, the planets do not even have a close approach to the 3:2 MMR. These results were expected from our conditions which are thus verified.

4 Summary

We study the orbital evolution of two super-Earths migrating in a protoplanetary disk using two-dimensional hydrodynamic simulations. We have demonstrated that convergent migration can be converted to divergent migration in a system of two super-Earths that are able to open partial gaps in the disk and thus the planets cannot become locked into the 3:2 MMR. Such behavior can be explained by the

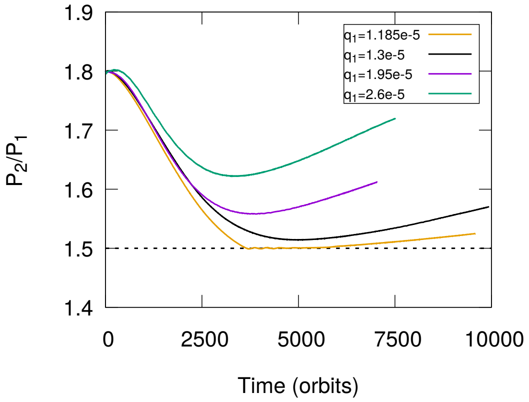

Fig. 3: Evolution of the period ratios for the simulations with two planets with $q_2 = 1.185 \times 10^{-5}$ and various values of q_1. These adopt the initial surface density given by Eq. 2.

wave planet repulsion mechanism. According to this angular momentum is transferred into the horseshoe region of a planet by density waves launched by the other planet and then to the planet itself through horseshoe drag. We give conditions for this mechanism to be effective derived from simple analytical considerations. These imply that planets need to form partial gaps in the disk, but enough material should remain in the coorbital region to allow the angular momentum to be transported to the associated planet. Our simulations confirm that if the conditions are satisfied for both planets, repulsion between them can take place. This prevents them being captured in resonances. We infer that this mechanism may account for the observational feature that planet pairs often have period ratios slightly in excess of strict commensurability, or deviate greatly.

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