Extended body corrections to the inverse square law for spherically symmetric sources

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The inverse-square law for calculating the irradiation arises as a direct consequence of the conservation of the energy, when spherical symmetry is imposed. The law implies that any spherically-symmetric source can be replaced, without changing the energy flux, by a point-sized source located in the center of symmetry. However, anybody who has seen a sunset knows, that when the center of the Sun moves slightly below the horizon, still a considerable portion of the stellar surface can be visible, irradiating much more than the corresponding point-sized source, which would be hidden under the horizon. This apparent contradiction is immediately solved when one realises that the presence of the planet, whose surface is absorbing part of the photon flux coming from the star, breaks the spherical symmetry, producing violations of the inverse-square in the sunset (or, equivalently, sunrise) region. On exoplanets that are extremely close to their star, this breakdown of the inverse-square law can become very important.

1 Introduction

The recent advances in the study of extra-solar planets have shed light on such planets that are very close to their parent stars (Mayor & Queloz, 1995). The irradiation received by these planets plays an important role in understanding the geological and climate properties of such planets. This underscores the importance of deriving a correct model for irradiation in such planets. The inverse-square (IS) law has been well established and has been used ubiquitously in previous studies (Charbonneau & Noyes, 2000). However, it doesn't suffice for extremely close planets.

2 Violation of the inverse square law

Considering a star-planet system, the blue region in Fig. 1 depicts the set of all those positions from which the star is not entirely visible. An observer (represented in our case by the eye) sitting on an arbitrary blue point would see a partially eclipsed star. Since in this case, some of the photons directed to the observer are screened by the planet, the flux fails to be radial. This opens the door to possible violations of the inverse-square law on the dark green region of the planet. The position at which the light-green region of the planet (where the inverse-square law is valid) meets the dark-green region (where the inverse-square law ceases to hold) is defined as the *critical belt of symmetry* and each point of it is a *critical point of symmetry*. Quantifying the importance of the violations of the inverse-square law in the dark-green region and their dependence on the geometrical parameters of the star-planet system is the main purpose of this work.

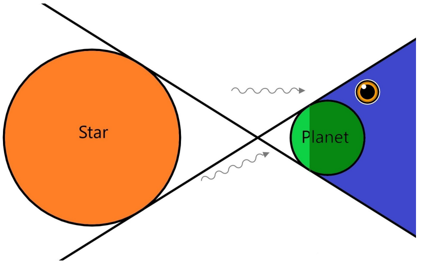

Fig. 1: Boundaries of validity for the inverse-square law.

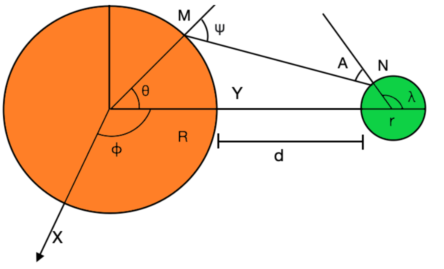

Fig. 2: Star-planet system geometry in spherical coordinates

3 Geometrical model

In our approach, we intend to integrate the irradiance from each surface element from the apparent surface of the star. For a close-in planet, the limits of integration are defined by the region bound between the tangents to the stellar circle from the point of observation on the planet. Additionally, the stellar surface hidden beneath the local horizon is omitted from the total flux received. Fig. 2 shows the system geometry with λ defining the latitude.

The analytical formulation of the new equation for the irradiance at a given latitude is written as follows:

$$\int dI = \int_{-\theta_\mathrm{n}}^{\theta_\mathrm{n}} \int_{-\theta_\mathrm{n}}^{\theta_\mathrm{n}} \frac{\sigma T_0^4 R^2 \cos\theta \cos A \cos\psi (1 - u(1-\mu)) d\theta d\phi}{\pi (d_\mathrm{s})^2}. \tag{1}$$

Here T_0 is the temperature at an optical depth of 1 and θ_n is the limit of stellar visibility that depends on a given latitude at the planet. Currently, the model assumes a linear limb darkening law, and therefore works best for Sun-like stars.

4 Results and Discussion

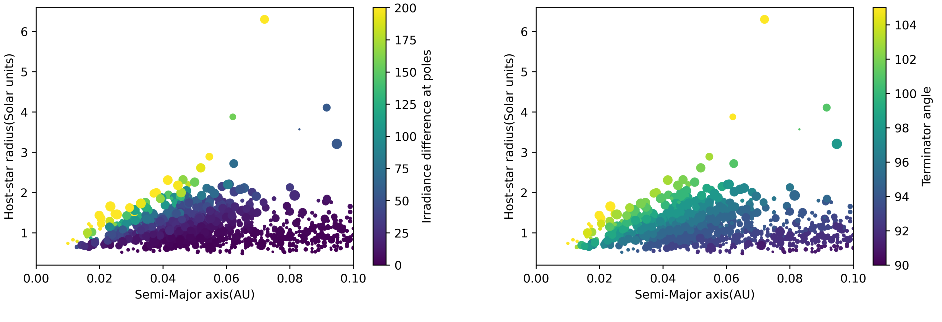

Fig. 3: Differences between the two approaches for a subset of the current the exoplanet population. The size of the points indicates the radius of the exoplanet. The terminator angle, for a given star planet system, determines how far will the terminator extend beyond the day-side of the planet.

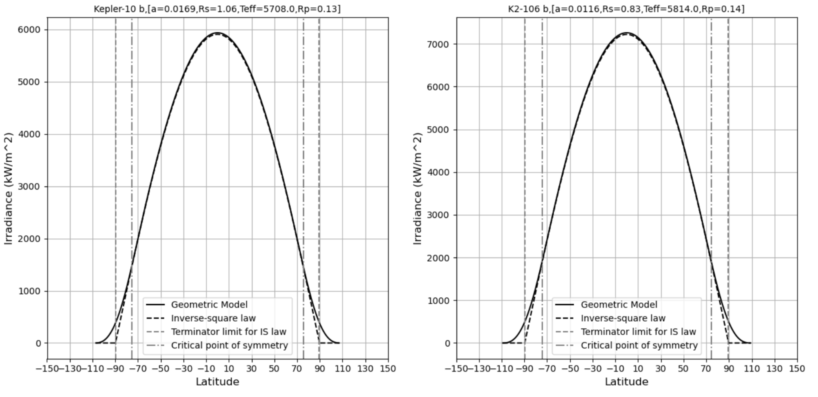

Fig. 4: Irradiance for Earth-like planets with stellar and planetary parameters. Semi-major axis is in AU and the planetary and stellar sizes are in Jupiter and solar radii respectively. The terminator limit for IS law represents the point after which the irradiance predicted by the law is zero and the critical point of symmetry shows the point after which we expect to see differences from the standard inverse-square calculation.

An important distinction between the two approaches comes at the poles of the exoplanet as seen in Fig. 3 and Fig. 4. Another interesting aspect of this model is the distribution of irradiation on the planetary hemisphere not facing the star. Unlike the IS model, the irradiance here extends towards the night-side of the exoplanet thus shifting the day-night terminator. It has been earlier proposed that even small changes in stellar irradiance may cause fundamental climatic changes on an Earth-sized planet, which in turn could affect its habitability (Forget & Leconte, 2014). Other studies (Nguyen et al., 2020) reproduce similar insolation patterns and hence consolidates our results. The openly available code *InstellCa*[1] uses the parameters from the extra-solar planet encyclopedia (Schneider et al., 2011) and the NASA exoplanet archive (Akeson et al., 2013) to compute the irradiance plots.

References

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[1]https://github.com/Mradumay137/InstellCa