A novel gaze on the Nicolaus Copernicus' *Commentariolus*

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The probable way of thinking of Nicolaus Copernicus' that lead him to a new concept of the Universe is studied in this paper. The information essential for the reconstruction of his reasoning process is provided in the Commentariolus. In this concise manuscript Copernicus, for the first time, outlined his innovative ideas. The time when the Commentariolus was written coincides with Copernicus' stay at the bishop's castle in Lidzbark Warmiński (Heilsberg). This fact turned out to be particularly interesting in the context of the recent discovery, which happened during the restoration of historical polychromes in the wall of the castle's cloister. Among over a hundred inscriptions found there was one thematically related to astronomy. This paper begins with the definition of simplified geocentric Ptolemy's model. Its subsequent analysis, in search of a coherent explanation of the second anomaly, results in the geostatic model of the planetary system, which may be considered as an intermediate step on the Copernicus way toward the heliocentric Universe.

1 Introduction

In his short manuscript known as the *Commentariolus*, and fully entitled *Nicolai Copernici de hypothesibus motuum coelestium a se constitutis commentariolus*, Copernicus comprises his thoughts on an alternative to Ptolemy's geocentric vision of the Universe. The exact moment when this work was written is not certain. It was limited by the dates of 1502, i.e. the publication of the known to Copernicus *Almanac,* and 1514 when a note about it appeared in the library inventory of Maciej of Miechów. However, it is probable that it was written before Copernicus left Lidzbark Warmiński in 1509 (Dobrzycki, 2013). Thanks to the copies of *Commentariolu*s circulating in Europe, Copernicus joined the scientific discourse of his time.

At the beginning of *Commentariolus*, Copernicus presents the current position of geocentric astronomy and provides a critical comparison of his ideas with the conventional Ptolemaic model of the Universe. Then, briefly, avoiding math, he outlines an early version of his heliocentric theory of the Universe. Aiming to find: *the more rational arrangement of wheels ... which would rotate uniformly with respect to its means, as required by the principle of perfect motion* (Kopernik, 2007). But, for all its innovativeness, Copernicus' theory retained the paradigm of uniform motion around the circle obeyed by all celestial bodies. Essentially, the planets move along the ecliptic from west to east, however, their motion is not uniform and shows two anomalies. The first anomaly, or the zodiacal anomaly, refers to the observed irregularity of the planetary velocity in their way around the ecliptic. It changes from its minimal value at the apogee to a maximal value at the perigee. The second anomaly, or solar anomaly, refers to the observed correlation of the current motion of the planet with the position of the Sun. The higher planets, orbiting above the Sun, make a retrograde loop in their way around the Earth when they are in the opposition to the Sun. On the other hand, the lower planets Venus and Mercury, are always close to the Sun. They also exhibit retrograde motion, but it happens when they are in the conjunction with the Sun, and hence their view is lost in the glare of the Sun. The Sun shows only the zodiacal anomaly, while the Moon does not exhibit retrograde motion at all (Pecker, 2001). The studies of planetary motions, undertaken to solve the problem of the first anomaly, guide Copernicus to the rigorous analysis of the second anomaly, and as a result, to the remarkable discovery. Unfortunately, he says nothing about how he arrived at this new model (Swerdlow, 1973). The purpose of this paper is to follow the thought process that led Copernicus to the foundations of the heliocentric concept of the planetary system.

2 The geostatic planetary model

The ancient astronomers quantitatively account not only for the regular motion of planets but also explained the zodiacal and solar anomalies and their mutual relationship. Hipparchus devised an eccentric. If the Earth is placed slightly off the center of the solar sphere, the Sun will continue to move around the center at a constant speed, but when viewed from an eccentric Earth, this motion will not be uniform. In turn, to explain the phenomenon of the retrograde motion of planets, he proposed a model of planetary deferents and epicycles. The planet circled a small circle – an epicycle whose center at that time was traveling in a large circle – deferent, around the center of the universe. The idea came down to decomposing the periodic function into the sum of simpler oscillatory functions. Through the appropriate selection of parameters: radii of circles and angular velocities, this system allowed to faithfully recreate the observed path of the planet in the celestial sphere (Gallavotti, 2001). Another geometric concept – *punctum aequans* – was introduced by Ptolemy. He placed this point on the opposite side of the center of different at the same distance as the Earth. This clever trick allowed the model's predictions to be better adapted to the observations and justified the changes in the planet's velocity, observed at various stages of its way over deferent while its motion remained uniform with respect to the equant (Evans, 1984).

The instantaneous position of the planet in the sky indicates the sum of two vectors: $R(t)$ pointing to the center of the epicycle and $r(t)$ showing the phase of the epicyclic motion of the planet. To trace the possible way of the transformation of a geocentric system into a heliocentric one, we apply the simplified Ptolemy model, assuming one common central point of the planetary system, neglecting the first anomalies of the planets, and ignoring their eccentricities $E = 0$. The last means the uniform motions of the centers of epicycles, as they are observed from the Earth, and that vector $R(t)$ has the constant length that equals the radius of the deferent $|R|$. Letting the function $\rho_{\mathrm{n}}(t)$ describe the position of the planet n in the complex plane at the time t, we have (Brown, 2005):

$$\rho_{\mathrm{n}}(t) = R_{\mathrm{n}}(\cos\Omega_{\mathrm{n}}t + i\sin\Omega_{\mathrm{n}}t) + r_{\mathrm{n}}(\cos\omega_{\mathrm{n}}t + i\sin\omega_{\mathrm{n}}t), \tag{1}$$

The right-hand side of Eq. 1 is a superposition of the motion of the planet in epicycle and the motion of the epicycle center over the deferent. The Ptolemy system

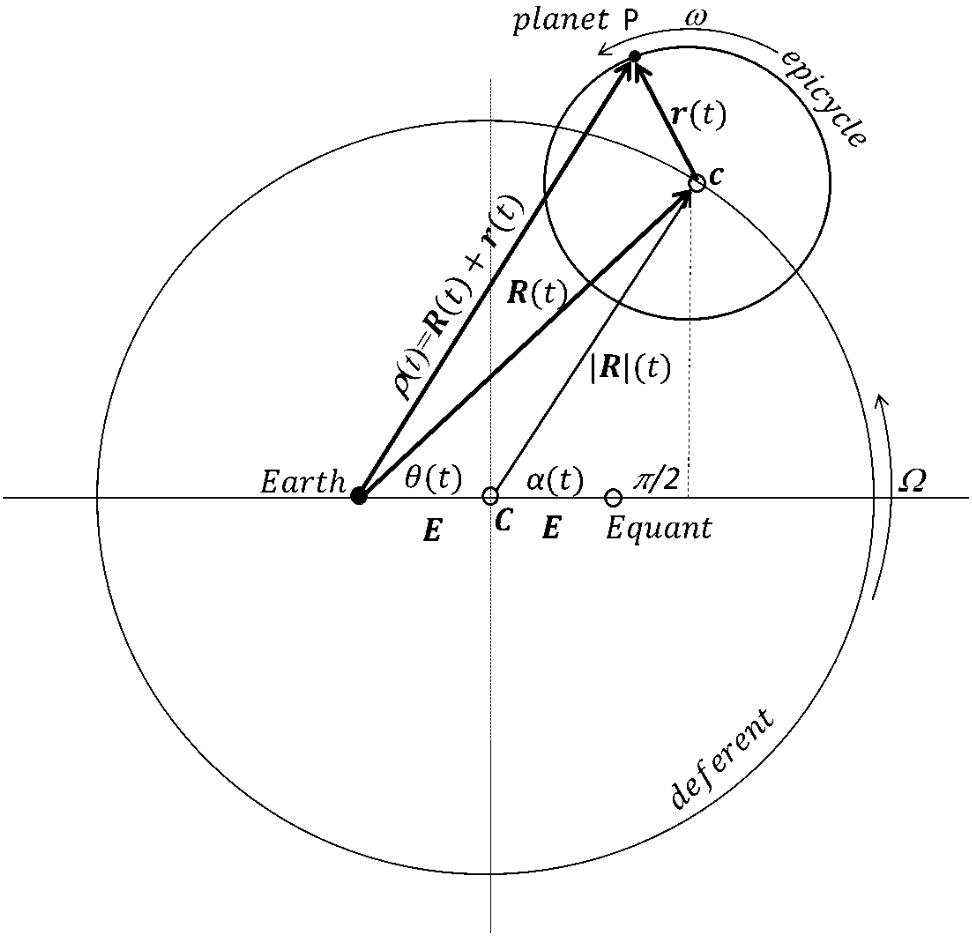

Fig. 1: The elements of Ptolemy's geocentric model of the planetary motion. The Earth and Equant are located eccentrically, E, relative to the center of deferent. The center of the epicycle moves along the deferent with a constant angular velocity Ω, while the planet circling the epicycle with velocity ω. In general, the angular velocity of the planet observed from the Earth is not constant.

described only the angular position of the planet in the firmament (Crombie, 1960). Similarly, the angular direction, defined by a complex number in Eq. 1, does not depend on its module. The distance R_{n} as irrelevant is only an arbitrary scaling factor. To sum up, the direction towards the planet n is completely determined by specifying the values of angular velocities Ω_{n}, ω_{n}, and the ratio $r_{\mathrm{n}}/R_{\mathrm{n}}$.

It is known that Copernicus was introduced to the secretes of Ptolemaic astronomy by reading the recently published work by Johannes Regiomontanus' *Epitoma in Almagestum Ptolemaei*, which is a summary and a critical analysis of Ptolemy's treatise (Regiomontanus, 1496). Regiomontanus drew attention to some unsatisfactory features and inconsistencies in Ptolemy's theory. There were no constraints imposed on scale factors of individual planetary orbits, as well as for the position of *superior* planets in the relation to the Sun. So, the centers of their epicycles could be located anywhere in the perimeters of their deferents. However, to ensure the retrograde motion happens at their opposition, Ptolemy imposed constraints on the spatial orientation of the radii of their epicycles. During the movement of the planetary system, the directions of these radii should remain parallel to the Earth-Sun direction. The motion of the *inferior* planets, always seen close to the Sun, was even more restricted. Their epicycles were rigidly fixed to the line connecting Earth with Sun. Then, even in the geocentric model Sun played a distinctive function (Fig.2).

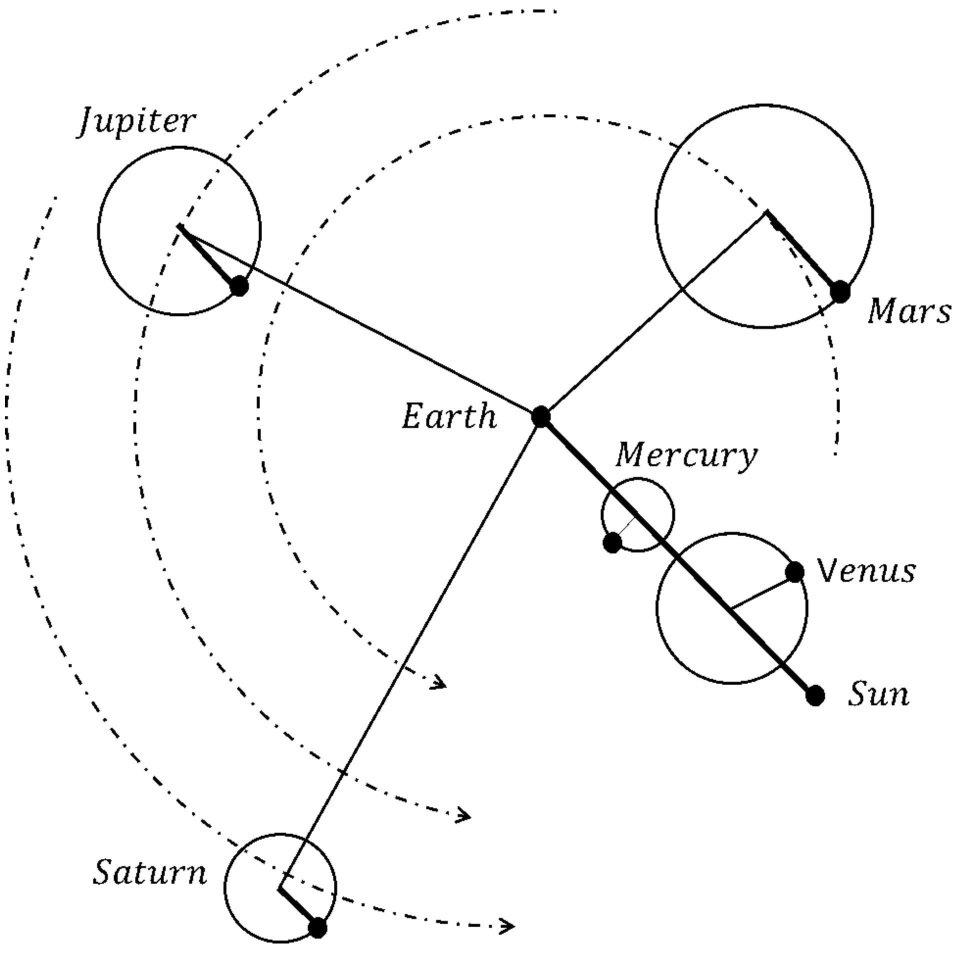

Fig. 2: The marked with a thicker line, the radii of deferents, of *inferior* planets and the Sun, as well as the radii of epicycles of *superior* planets, remain parallel to each other during the motion of the whole geocentric system (Zeilik, 2002).

The postulated by Ptolemy synchronization of the motion of the deferents' radii of *inferior* planets and epicycles' radii of *superior* planets, with the position of the Sun, reveals the unexplained, in his model, a connection between some angular speeds of the planets. They obey the following relationship:

$$\Omega_0 = \Omega_1 = \Omega_2 = \Omega_3 = \omega_4 = \omega_5 = \omega_6, \tag{2}$$

where the common value $\Omega_0 = 2\pi/1\text{year}$, and the numbering of individual terms indicates the sequence of planets starting from the stationary Earth, 0, and ending with Saturn, 6. The ancient astronomers respected a rule that the larger of the two circles is a deferent, and the smaller is an epicycle. However, as shown in Fig.3, the resultant vector $\rho_\mathrm{n}(t) = R_\mathrm{n}(t) + r_\mathrm{n}(t)$ being the sum of two components is independent of their order. And so, switching the deferent and the epicycle doesn't affect the motion of the planet. During the orbital motion of superior planets, the direction from Earth toward the center of the large circle turns out to be steadily pointing towards the mean Sun, S.

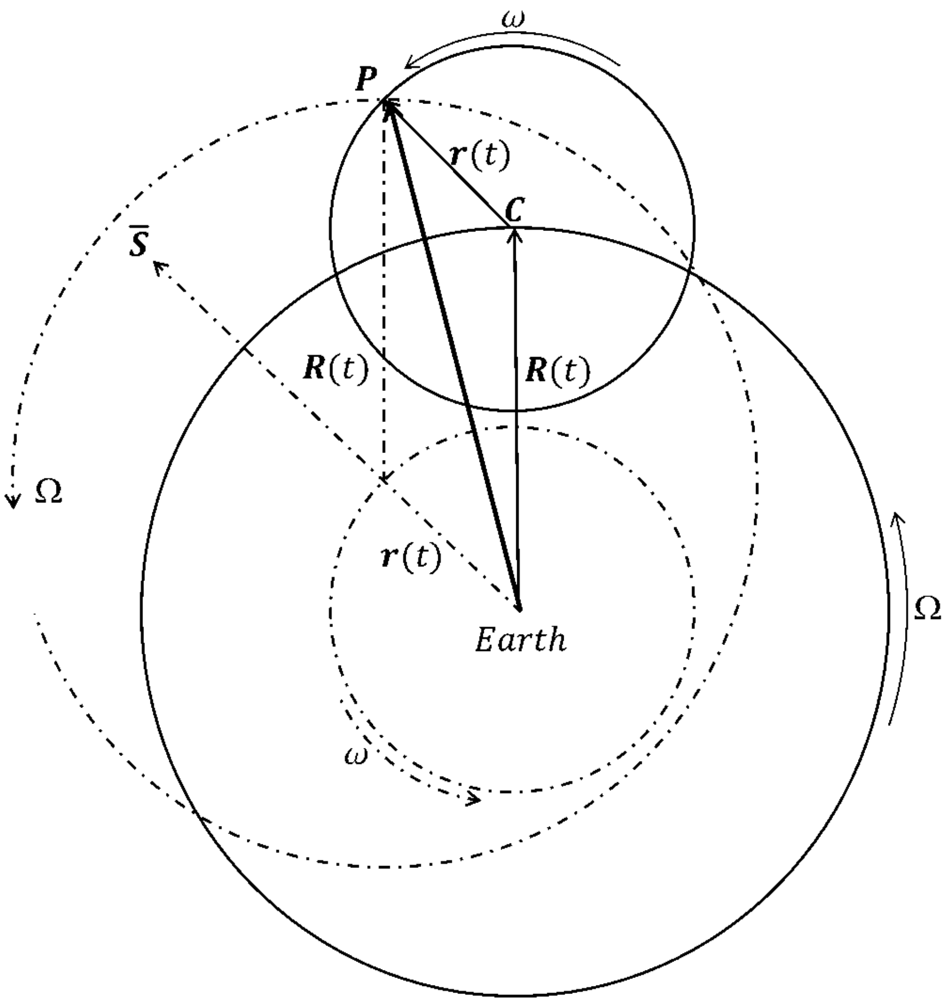

Fig. 3: An alternative description of the superior planet's motion in the geocentric system (Goldstein, 2002).

Regiomontanus proposed a similar transformation also for the inferior planets. It made it possible to build a logically coherent planetary system by introducing in Eq. 1 the scale factors R_n such that the terms with the same angular velocities Ω_0 (Eq. 2) all have the same radii. Thus, for some value R_0 we require:

$$R_0 = R_1 = R_2 = R_3 = r_4 = r_5 = r_6, \tag{3}$$

Fig. 4: The inscription engraved with a compass, discovered on the wall of the cloister of the bishops' castle in Lidzbark Warmiński, depicting an image of concentric circles crossed by an eccentric circle (Photo courtesy Museum of Warmia and Mazury).

On imposing the conditions in Eqs 2 and 3, and taking the Earth-Sun distance R_0 as the unit, the equations corresponding to the locations of the planets have the

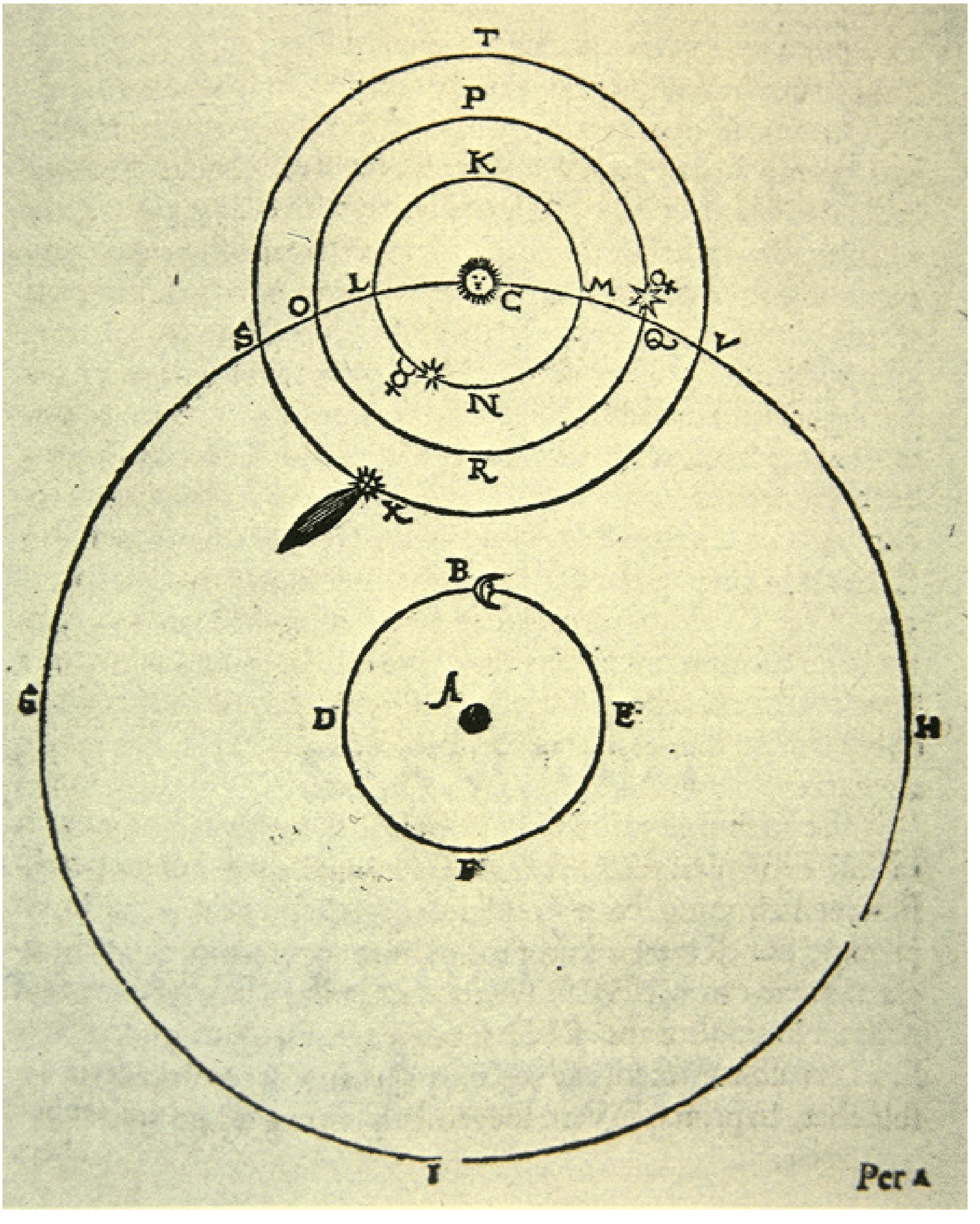

Fig. 5: The model of the planetary system of Tycho Brahe with an image of a comet from 1577 (Public domain).

following form:

$$\lambda_0(t) = 0, \tag{4}$$

$$\lambda_1(t) = e^{i\Omega_0 t} + \frac{r_1}{R_0} e^{i\omega_1 t}, \tag{5}$$

$$\lambda_2(t) = e^{i\Omega_0 t} + \frac{r_2}{R_0} e^{i\omega_2 t}, \tag{6}$$

$$\lambda_3(t) = e^{i\Omega_0 t} + \frac{r_3}{R_0} e^{i\omega_3 t}, \tag{7}$$

$$\lambda_4(t) = e^{i\Omega_0 t} + \frac{r_4}{R_0} e^{i\omega_4 t}, \tag{8}$$

$$\lambda_5(t) = e^{i\Omega_0 t} + \frac{r_5}{R_0} e^{i\omega_5 t}, \tag{9}$$

$$\lambda_6(t) = e^{i\Omega_0 t} + \frac{r_6}{R_0} e^{i\omega_6 t}. \tag{10}$$

The sequence of equations begins with stationary Earth, through Mercury, Venus, Sun, and planets in super solar orbits, like Mars, Jupiter, and Saturn. The motion of the planetary system is, in Eqs 4 - 10, described by two functions of time. The first is specific to a given planet, and the second is common for all of them. The system of equations represents a geostatic model of a planetary system in which the planets, orbiting the Sun and with it, orbit the Earth for one year. There is no hard evidence that Copernicus considered the geostatic system as an intermediate step, in his path toward heliocentrism but it seems to be a natural approach (Gingerich, 2002).

3 Conclusion

The enigma of Copernicus's pathway to the truth was recently recalled by an accidental discovery.

In 2011, during the renovation works of 14th-century wall polychromes on the first floor of the cloisters of the bishop's castle in Lidzbark Warmiński, more than a hundred historical inscriptions scratched in the plaster were discovered. Among them, at the entrance to the one chamber, a scheme showing five (or six?) concentric circles intersected by another eccentric circle was found Fig.4 (Kozarzewski et al., 2016).

This drawing, carefully carved with a compass, strikingly resembles Brahe's geostatic system Fig.5 (Brahe, 1588). The time of this engraving creation, its location, and its form might indicate its author. Copernicus does not mention when he made his crucial discovery and does not explain how it came to him. Yet, it is remarkably probable that it happened during his stay at the castle in Lidzbark.

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