Halo Mass Function as a Probe of Gravity

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We have studied the halo mass function (HMF) in two modified gravity (MG) models: the Hu-Sawicki $f(R)$ gravity and the normal branch of Dvali-Gabadadze-Porrati (nDGP) models. We found that when we express MG HMF as a function of $\ln(\sigma^{-1})$, it approximates a universal shape across redshifts. This result has already been established for ΛCDM. Furthermore, we found a systematic trend in the deviation of the mass function obtained for these models when we compare with the ΛCDM results. We exploited this property to devise a method to obtain analytical expressions for MG HMF, which, in turn would save us the need to run dedicated MG simulations to compute this cosmological measure.

1 Introduction

The current ΛCDM model of our universe, based on the Einstein's theory of general relativity (GR), successfully explains the evolution of our universe and has passed many tests (Abbott et al., 2016; Planck Collaboration, 2020; Alam et al., 2021a) which, time and again, have proven its credibility across length scales and cosmic epochs. However, owing to its phenomenological nature and many observational and theoretical challenges (Weinberg, 1989; Bullock & Boylan-Kolchin, 2017), the model has been subject to scrutiny since its inception and a plethora of modifications to the standard GR theory have been proposed as alternatives to the standard ΛCDM. These modifications are expected to leave signatures on the cosmic observables, and in particular on the underlying properties of the density field, when compared with the standard case. In this work, we focus on the study of one such measure, termed the *halo mass function* (HMF).

1.1 Halo mass function

Dark matter clusters into gravitationally bound structures, termed as *halos*, and it is within the gravitational potential well of these dark matter halos that baryons cool and condense to form galaxies and the visible large-scale structures (LSS). Hence, the abundance of halos across redshifts is an important statistical quantity in cosmology as the abundance is associated with the dynamics of LSS formation and evolution. It helps to probe the underlying theory of gravity that governs the evolution of perturbations and the properties of LSS.

This abundance of halos is quantified in terms of the number density of halos within a logarithmic mass interval, $dn/d\ln M$, and is referred to as the HMF. The first modern formalism for HMF was provided in the seminal work of Press & Schechter (1974) in which, for a halo of mass M, HMF is given by:

$$\frac{dn}{d\ln M} = \frac{\rho_{\rm m}}{M} F(\sigma) \left| \frac{d\ln\sigma}{d\ln M} \right| . \tag{1}$$

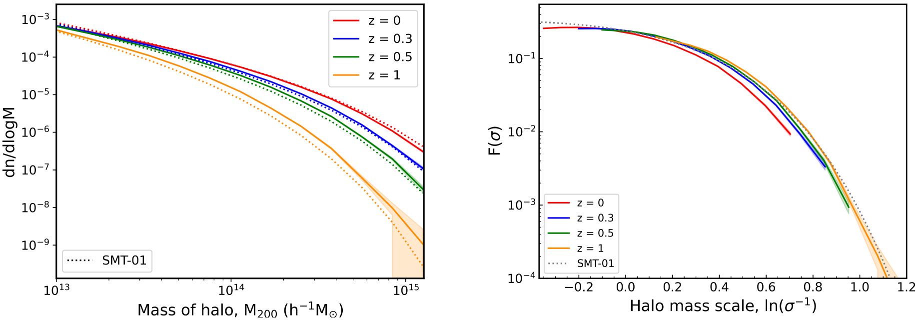

Fig. 1: *Left*: ΛCDM halo mass function (HMF), plotted as a function of the halo mass M_{200}, across redshifts $z = 0, 0.3, 0.5$ and 1. Dotted lines are the theoretical HMF predictions of Sheth et al. (2001). *Right*: Halo multiplicity function $F(\sigma)$, plotted as a function of $\ln(\sigma^{-1})$. Here, we can acknowledge that with proper re-scaling, the HMF can be expressed approximately as a single universal curve across redshifts.

Here $\rho_{\rm m}$ is the matter density, $F(\sigma)$ is the *halo multiplicity function*, while $\sigma(z, R)$ is the variance in the density fluctuation field smoothed using a top-hat filter of scale $R = (3M/4\pi\rho_{\rm m})^{1/3}$, extrapolated to the redshifts under study, and is given by:

$$\sigma^2(R, z) = \frac{1}{2\pi^2}\int_0^\infty k^2 P(k, z) W^2(kR) dk. \tag{2}$$

Here $P(k, z)$ is the linear theory matter power spectrum and $W(kR)$ is the Fourier transform of the top-hat window function.

1.2 Universality of the halo mass function

It is a well-known result that if we express $F(\sigma)$ as a function of $\ln(\sigma^{-1})$ rather than of the halo mass, HMF takes a universal form across redshifts (Jenkins et al., 2001). In the right plot of Fig. 1, we can see that after expressing length scales in terms of $\ln(\sigma^{-1})$, the redshift dependence seen in the left plot is approximately eliminated. Here, the halo mass can be substituted with $\ln(\sigma^{-1})$ as these two quantities have a redshift-dependent monotonic relation, and hence the σ term encapsulates all the redshift dependence.

This universal property of HMF has motivated many authors to propose empirical fits for this function, and the parameters of these expressions are calibrated using N-body simulations. This approach has proven to be highly advantageous as it saves the time of running simulations and gives a reasonable estimate of HMF across redshifts, length scales, and cosmologies. In our work, we have used one of these empirical predictions proposed by Sheth et al. (2001).

2 N-body simulations and modified gravity models

We have used dark matter halo catalogs generated using ELEPHANT (Extended LEnsing PHysics using ANalaytic ray Tracing) cosmological simulations (Alam et al., 2021b), which consider ΛCDM, two variants of the Hu & Sawicki (2007) $f(R)$ gravity model with the value of the model parameter, $|f_{R0}| = 10^{-5}$ and 10^{-6} (decreasing order of deviation from ΛCDM), which are here referred to as $f5$ and $f6$, respectively, and two variants of the normal branch of the DGP gravity model (nDGP; Dvali et al., 2000) with the model parameter $r_c H_0 = 1$ and 5 (again in decreasing order of departure from ΛCDM), denoted as N1 and N5, respectively. All the gravity models are simulated against the background parameters consistent with the WMAP9 cosmology (Hinshaw et al., 2013) and the data are analysed from the snapshots generated at $z = 0, 0.3, 0.5$, and 1.

Screening mechanisms are inherent to MG models as they are essential to restore GR in the regimes where it has been well-tested. The chameleon screening (Khoury & Weltman, 2004) is employed for the $f(R)$ gravity scenarios, while the Vainshtein (1972) mechanism screens the fifth-force for the case of nDGP models.

3 Results

3.1 Halo mass function in modified gravity cosmologies

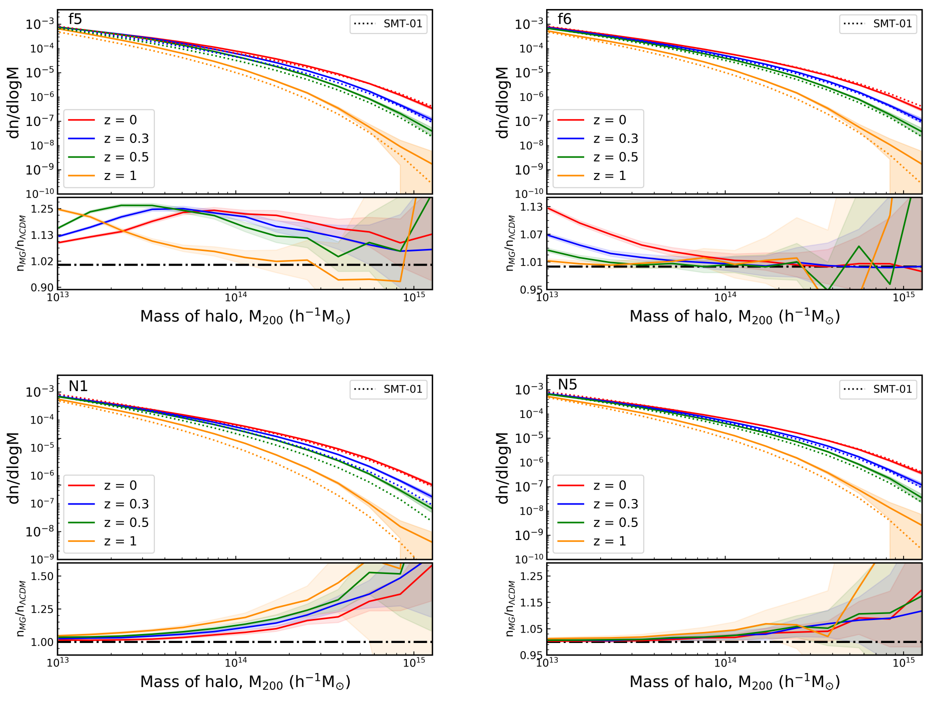

Fig. 2: *Top sub-panels*: Modified gravity halo mass function plotted as a function of halo mass. Each colour corresponds to a redshift indicated in the legend. Dotted lines are the theoretical HMF predictions based on Sheth et al. (2001). *Bottom sub-panels*: Ratio of the MG HMF to the ΛCDM one for each redshift.

The introduction of new physical degrees of freedom in these MG scenarios impacts the structure formation dynamics, which would, in turn, lead to a different evolution of perturbations than what we would expect from the ΛCDM. This is expected to impact the abundance of collapsed structures and as a result, the HMF.

In the top sub-panels of Fig. 2, we have plotted how MG HMF scales with halo mass (expressed as M_{200}) at different epochs. Their ratio w.r.t. the ΛCDM, shown in the bottom sub-panels clearly illustrates that for all the models at all redshift, the value is different than 1, thus,indicating a change in the HMF predictions for MG models w.r.t. the standard ΛCDM case.

The $f(R)$ gravity models undergo environmental and self-screening in low and high-mass regimes, respectively, and hence, we would expect a resultant peak-like feature in the ratio at the intermediate-mass scales (given $M \propto \sigma^{-1}$), where both the location and the amplitude of the peak depends upon the strength of the fifth force. For the $f5$, the peak is at a much larger mass (or smaller σ) scale and is of greater amplitude compared to the $f6$. Also, the location of the peak is dependent on the redshift. The peak is at the largest mass scale for $z = 0$ and at the smallest scale for $z = 1$. This can be explained by the fact that as time passes, the amplitude of the fluctuation density field increases, thereby increasing the scales of chameleon screening, and shifting the peak to larger scales.

For nDGP models, we see a monotonic enhancement in the HMF with mass, which is greater in the N1 compared to the N5. This enhancement is more pronounced at the large mass end where the sizes of the halos are much greater than their Vainshtein radius and the fifth force is unscreened, thus, larger structures grow more by accreting more matter from regimes not affected by screening. Again, we see an explicit dependence on redshift. The amplitude of enhancement decreases as time passes as the Vainshtein radius for a given mass increases, thereby increasing the screening at later times.

3.2 Re-scaling $F(\sigma)$ in modified gravity models

As discussed above, instead of the halo mass we can use density fluctuations, given by $\sigma(R, z)$, to express the HMF. This substitution captures the redshift dependence of the HMF and leads to a universal form of the mass function across cosmic epochs. In Fig. 3, we use the same data as in Fig. 2, but this time illustrated in terms of the $F(\sigma)$ - $\ln(\sigma^{-1})$ relation. The top sub-panels show this relation for the considered MG models across all redshift ranges and we observe that they all exhibit a similar, universal trend in the multiplicity function, as is known for the case of ΛCDM (right plot of Fig. 1). From this, we conclude that the mass function can be approximately expressed as a single curve, even for the case of these two MG models, despite the inherent non-linear screening and scale-dependent growth enhancement in them. This can be attributed to the encapsulation of the MG-induced effects in the changes of the $\sigma(M)$ term, which results from the changes in the growth rate in the MG models compared to the ΛCDM.

In the bottom sub-panels of Fig. 3 we found that when we focus on the ratio between the MG $F(\sigma)$ and the ΛCDM one, we again obtain a universal curve. Namely, the explicit dependence of the deviation in the mass function on redshift is eradicated in these re-scaled plots. For the case of nDGP gravity, we needed to perform an additional re-scaling of the matter variance to get $\ln(\widetilde{\sigma}^{-1})$. For details of this re-scaling, refer to Gupta et al. (2022).

Owing to this universal property, we can express MG HMF as a functional deviation from the ΛCDM results given as:

$$F(\sigma)_{\mathrm{MG}} = \Delta_{\mathrm{MG}} \times F(\sigma)_{\Lambda\mathrm{CDM}}. \tag{3}$$

Here, $F(\sigma)_{\Lambda\mathrm{CDM}}$ is the HMF obtained from the ΛCDM simulations. Δ_{MG} are expressed as simple analytical functions, form of which depends on the MG model considered and are calibrated for the universal deviation, $\frac{F(\sigma)_{\mathrm{MG}}}{F(\sigma)_{\Lambda\mathrm{CDM}}}$ from ELEPHANT simulation results. The details of the functional forms for Δ_{MG} and the tests on our proposed fitting can be found in Gupta et al. (2022).

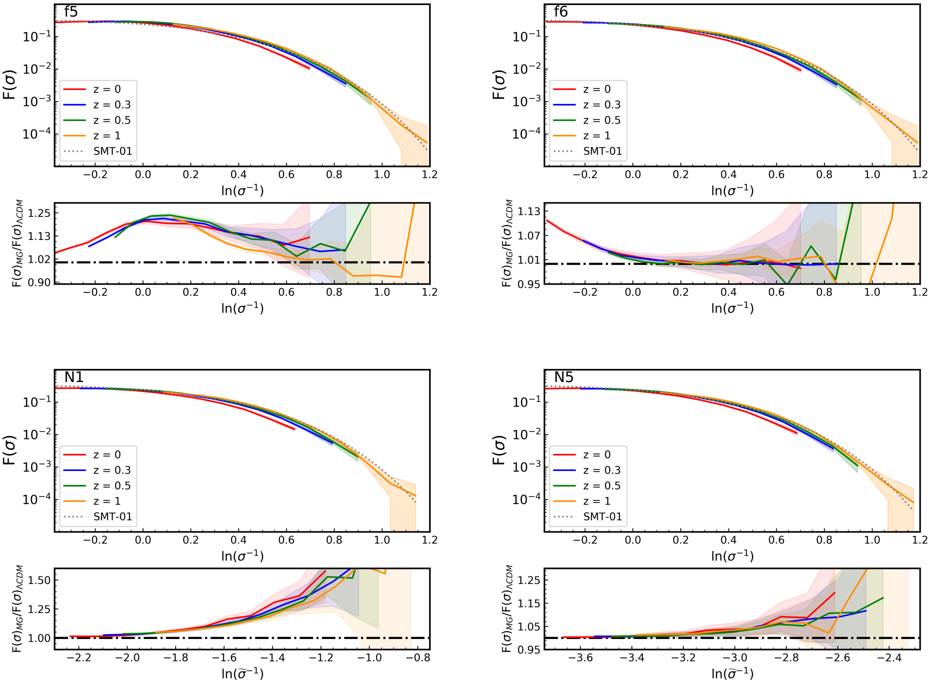

Fig. 3: *Top sub-panels:* Halo multiplicity function $F(\sigma)$ plotted as a function of $\ln(\sigma^{-1})$. Each colour corresponds to a redshift indicated in the legend. Dotted lines are the theoretical HMF prediction based on Sheth et al. (2001). *Bottom sub-panels:* Ratio of MG $F(\sigma)$ to the ΛCDM one for each redshift.

4 Summary and conclusions

We investigated how the HMF behaves in two MG models: $f(R)$ and nDGP gravity, using results generated from ELEPHANT N-body simulations.

HMF shows an explicit dependence on redshift when analyzed as a function of halo mass. However, when the HMF is expressed in terms of the density field variance, $\ln(\sigma^{-1})$, it assumes a universal time-independent trend and can be approximated as a single curve. This has been known for the ΛCDM case, but we have also found similar approximately universal behavior for the MG models that we have studied.

We focused on the systematic deviation in the MG HMF w.r.t. the ΛCDM. Similarly as for the absolute HMF values, this departure has an explicit dependence on redshift, when expressed as a function of halo mass. However, when expressed in the units of $\ln(\sigma^{-1})$, we can capture this redshift dependence and obtain a universal curve for this deviation of mass function. This has further helped us to obtain analytical approximations for the MG HMF, using only the ΛCDM N-body simulation results. This is advantageous as unlike in the ΛCDM, the N-body simulations for beyond-GR scenarios implementing fully non-linear treatment of the associated scalar fields and screening mechanisms are extremely expensive and time-consuming. Also, considering the ΛCDM simulations ensures that the non-linear effects associated with the structure formation dynamics are incorporated to the limit of the considered runs.

The formulation of the MG HMF is our first step towards semi-analytical modeling of large-scale structures in beyond-GR models. Given the considerable amount of computational resources needed to run MG N-body simulations, the development of semi-analytical models is important to generate quicker estimates for large-scale structures in the MG scenarios and precise interpretation of data from surveys.

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