

Conceptual foundations and issues in cosmology

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In these lectures I discuss some fundamental conceptual problems of cosmology, which are usually omitted in the standard courses, though they significantly influence our understanding of the Universe. These include the very notion of the Universe and consequences of its uniqueness, applicability of physical quantities such as distance and velocity in cosmology, the issue of various fine tunings and consequences of that the Universe had its origin in the initial Big Bang singularity. The fundamental features of the Universe were determined soon after the Big Bang and this epoch is hidden under the *physics horizon* and practically inaccessible to us. Cosmology is disparate from other physical sciences.

1 Introduction

“Cosmology is the attempt to understand in scientific terms the structure and evolution of the universe as a whole.” This sentence, opening an introductory article by H. Zinkernagel to the Special Issue on Philosophy of Cosmology of *Studies in History and Philosophy of Modern Physics* 46 (Zinkernagel, 2014), in its attempt at giving, in a very concise form, the aim of cosmology, sounds quite innocent since the formulated aim does not reveal to what extent cosmology is distinct from all other natural sciences. Yet the articles filling that Special Issue of the journal have clearly shown conceptual, epistemological, methodological and even metaphysical issues in modern cosmology. Most cosmologists underestimate or simply ignore the philosophical aspects of their field, they work in it as if they have been dealing with astronomy and physics proper and they are successful in achieving their outcomes and publishing them. Apparently they are right since cosmology is frequently identified with *cosmophysics*, the latter being a union of theoretical physics applied to the Universe and astronomy (observations of distant systems of very large objects.) Nevertheless one need not read the mentioned volume on philosophy in cosmology to recognize without any doubt that like no other science is cosmology based on philosophy and generates philosophical issues.

A simple example: in all natural sciences one has a sharp distinction between the external observer and the object under investigation. In cosmology it is impossible since we are just a small part of the object. We cannot look at the Universe from outside. This problem is well known in social sciences and humanities and distorts them: whereas the scholar dealing with a remote history is capable of being objective and uninfluenced by emotions and various prejudices, the scholar studying present-day society encounters strong psychological challenges as its member. Physicists claim that their field is more reliable than social sciences due to, among other reasons, the non-existence of these problems. But this is not the case of cosmology.

The core issues are:

1. What constitutes an explanation in cosmology?
2. How do we test validity of our explanations?
3. Are notions taken from physics adequate to cosmology?
4. Is the knowledge of the Universe as a whole as firm as is the laboratory physics (or at least as astronomy of our not very distant surroundings)?

The first three issues require a detailed discussion. A general and quite noncontroversial answer to the last issue is that our knowledge of the Universe as a whole is “softer” than astrophysics and there are *cosmological myths* adequate for our time.

The literature on these issues published in the last twenty years is quite abundant, showing that more and more scientists have been interested in the foundations of cosmology. This provides not only a deeper understanding of the Universe, but also has practical consequences: the deeper understanding indicates that some investigations in the very near past were based on some misconceptions. In preparing these lectures I have benefitted a lot from articles by Zinkernagel (2014); Ellis (2007, 2014); Kragh (2014); Butterfield (2014); Lopez-Corredoira (2014); Brandenberger (2014); Smeenk (2013, 2014); Goenner (2009) and my opinions on the problem of the uniqueness of the Universe are heavily based on papers by the most eminent expert in the field (Ellis, 2007, 2014). I recommend all these articles to the interested reader.

2 Cosmology as a mathematized historical science

The power of physics lies in that it precisely predicts the future, both passively (motion of a planet around the Sun, evolution of a star) and actively (the work of any mechanical or electronic device). Yet the Universe is observed at large space and time and is measured on and inside the *past light cone*, and the vertices of these cones cover a tiny part of the Earth worldline. Clearly recent observations reach far deeper into space and are richer in their variety than they were in previous centuries, nevertheless we can get from them far less information about the Universe than we can learn about various physical systems around us by applying laboratory physics. Experiments cannot be carried out in cosmology. This obvious statement (bordering on triviality) is frequently forgotten in theoretical considerations and we shall repeat it several times in these lectures.

Observational cosmology is analogous to geology, paleontology, archeology and political and economic history in that it deals with the past. It differs from them in that there are mathematical models of the Universe (and they are definitely more reliable than mathematical models in economy) and these make the history of the Universe more precise than the geological history of the Earth or human history. History as such has two aspects. The history of each nation, country, continent, mountain chain or evolution of animal species is *unique* — it is different from all other histories of similar objects. On the other hand, these differences are not absolute and there are many deep similarities between histories of distinct nations, or continents, mountains or groups of animal species. These similarities allow one to make general statements about human history and evolution in geography and biology. Without these “similarities in diversity” history would be devoid of any meaning.

Yet the Universe is *unique* and there is nothing to be compared with it, no diversities and no similarities. I stress that in physics proper, predictions mean foretelling

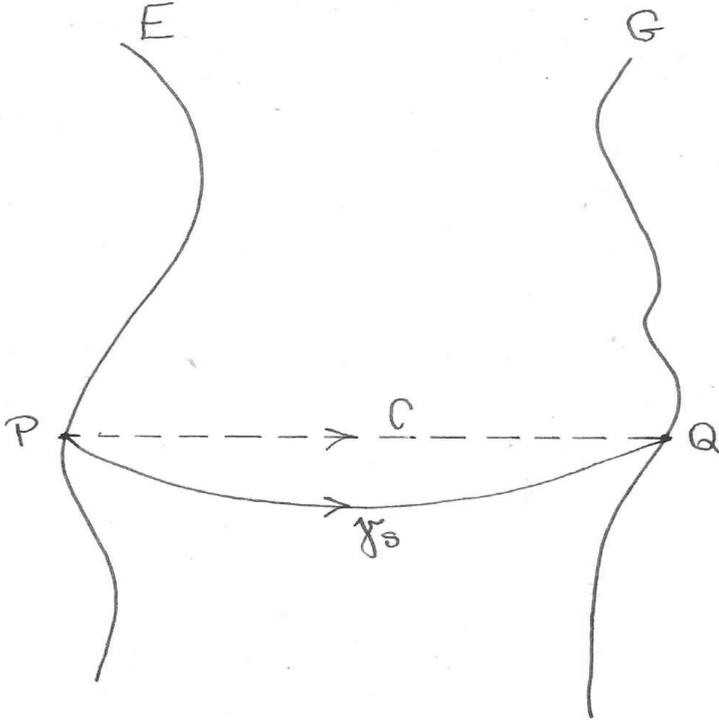


Fig. 1: In Robertson–Walker spacetime the distance between Earth E and Galaxy G is the length of the shortest spacelike curve joining simultaneous (at present) points P and Q on their worldlines. This curve is the unique radial spacelike geodesic from P to Q . In an expanding universe the curve C comprised of simultaneous points is longer than γ_s comprised of points earlier than P and Q . γ_s is the geodesic determining the distance between P and Q . Its length depends on the unknown radial coordinate of Q .

future states from the present state of some object. Of course, in cosmology one can also predict the future — the famous Friedmann diagram shows the future of the three Friedmann spacetime models (spatially flat, open and closed) and this diagram is an inevitable part of any course on cosmology. It is however clear that these predictions of the cosmological future are pointless, since they concern what will occur according to the theory in a billion years or later. Such predictions cannot be verified. Cosmology must be and is directed to the past and a *prediction* means a conclusion from the theory and present observations to *past* times — “cosmologists are prophets for the past” (Goenner, 2009).

Cosmology is a mathematized history of the Universe, a sequence of facts of its past which are logically ordered and quantitatively described in the language of physics and mathematics and these are expressed with high rigour. This sequence cannot be compared to another sequence and in this sense our understanding of the Universe is very limited. I shall return to this statement a couple of times.

3 The issue of cosmological distances

Now I return to the core issue no. 3: are the notions taken from physics adequate for cosmology? This is the issue of to what extent is physics adequate to describing the Universe? Two concrete questions:

What is the distance to a remote galaxy? And what is its velocity relative to us?

In this section I deal with cosmological distances. Entire books and numerous papers are devoted to methods of *measuring* great distances (“the ladder of cosmological distances”), but very little space is devoted in them to the very notion — what is distance?

In laboratory physics the distance between points in space and events in spacetime is well defined conceptually and is reliably measured. Problems appear in astronomy. Within the solar system distances are measured by the radar echo method and this method tells us the distance to, say, Mars at the moment of reflection of the signal by Mars. Within the Milky Way we assume a flat spacetime geometry and we may measure distances defined by a measurement method: paralactic distance, luminosity distance, angular diameter distance etc. In Euclidean space all these distances coincide. A real and actually unsurmountable difficulty arises when we deal with cosmology in a time dependent curved spacetime. How to define there a distance to a remote galaxy?

The Earth E and the galaxy G are represented in spacetime by worldlines and a distance between them must be defined as a distance between a point P on the worldline E and a point Q on the worldline G, see Fig. 1. As P we choose the present instant, yet in an arbitrary curved spacetime there is no “natural” (that is, geometric) way of choosing Q, it is arbitrary. At first sight cosmology is in a lucky situation in this respect, since in the, commonly used in cosmology, Friedmann models (based on the Robertson–Walker geometry) there is a distinguished time coordinate — the *cosmic time* measured by the *fundamental observers* — those for which the space at each instant of this time is homogeneous and isotropic. (These observers stay at rest relative to the centers of large galaxy clusters, or more precisely, at rest relative to the cosmic microwave background relic radiation.) Using this time, which provides a foliation of the cosmological spacetime, we naturally choose Q as simultaneous with P. Unfortunately this helps little.

According to spacetime geometry the distance between P and Q is the length of the shortest spacelike geodesic line connecting P and Q. In the Robertson–Walker geometry there is only one such geodesic line. The spacetime interval in this geometry is

$$ds^2 = c^2 dt^2 - R^2(t)[dr^2 + f_k^2(r)(d\theta^2 + \sin^2\theta d\phi^2)],$$

$$f_k(r) = \begin{cases} r & \text{for } k = 0 \\ \sin r & \text{for } k = +1 \\ \sinh r & \text{for } k = -1 \end{cases}$$

and $R(t)$ is the cosmic scale factor. Then defining the square of the length of a spacelike curve as $d\sigma^2 = -ds^2 > 0$, the distance from P to Q is the length of the shortest curve — the spacelike geodesic γ_s joining P and Q,

$$d_g(P, Q) \equiv \int_{\gamma_s} d\sigma.$$

The coordinates are $(x^\alpha) = (ct, r, \theta, \phi)$, $x^\alpha = x^\alpha(\sigma)$ and the tangent vector to the

geodesic is $k^\alpha = \frac{dx^\alpha}{d\sigma}$. Any spacelike geodesic satisfies the *geodesic equation*

$$\frac{d^2x^\alpha}{d\sigma^2} + \Gamma_{\mu\nu}^\alpha \frac{dx^\mu}{d\sigma} \frac{dx^\nu}{d\sigma} = 0,$$

where $\Gamma_{\mu\nu}^\alpha$ are the Christoffel symbols for the spacetime metric $g_{\mu\nu}$. We search for the solution to this equation describing a curve joining P and Q. Here one meets two problems. Firstly, one usually solves the nonlinear geodesic equation assuming the initial data: the initial point P and a direction of the geodesic, the tangent vector $k^\alpha(P)$. Yet here one has given two endpoints of the geodesic. Let us try to guess the solution. The space is homogeneous and spherically symmetric, hence we locate the origin of spatial coordinates (r, θ, ϕ) on the Earth's worldline and this implies that the geodesic is a radial line, θ and ϕ are constant along it. Now we guess: the geodesic is the radial curve C of simultaneous points: $x^0 = \text{const} = ct_P$, $r = F(\sigma)$, $\theta = \pi/2$, $\phi = 0$ with unknown $F(\sigma)$ to be determined by the geodesic equation. It is quite astonishing that C is NOT a geodesic! (This is a frequent error.) This outcome can be easily explained. Let $R(t)$ be an increasing function, as it is in the real world. Take two nearby points on a radial line emanating from P. On C their distance is given by $d\sigma^2(C) = R^2(t_P)dr^2$. Now take a radial curve γ_s joining P and Q as in Fig. 1. The distance of two nearby points on it satisfies $d\sigma^2(\gamma_s) = -c^2dt^2 + R^2(t)dr^2$. Along it $t < t_P$ and then $R(t) < R(t_P)$, what implies $d\sigma^2(\gamma_s) < d\sigma^2(C)$ and γ_s is shorter than C. The curve C does not determine the geometric distance, nevertheless it is singled out by its simplicity and its length between P and Q is called the *proper distance*

$$d_p(P, Q) = R(t_P) \int_P^Q dr.$$

One might find the geodesic γ_s and calculate its length from P to Q (I have never encountered the solution in the literature), but it would be useless. For here we meet the second difficulty, which cannot be overcome in cosmology. The radial coordinate of P is $r_P = 0$ and of Q is some $r_Q > 0$. The proper distance is then $d_p(P, Q) = R(t_P)r_Q$ and the geometric (geodesic) distance is $d_g(P, Q) = d_g(r_Q)$ with some function $d_g(r)$. But r_Q is unknown! In mathematics it is assumed that each point of a space may be given coordinates and the metric tensor is postulated, then distances may be calculated. In the physical world coordinates (in any coordinate system) may be assigned to space points if these points are marked in a rigid solid body and the body is macroscopic of "laboratory" size (not larger than the Earth). I can mark with a piece of chalk a number of points on the blackboard and determine their mutual distances using the flat geometry. Astronomers can assign angular coordinates on the celestial sphere to various objects (planets, stars, quasars) applying the "fixed stars", but their radial coordinate is unknown and may be determined only by measuring the distance to them. We conclude: both d_g, d_p are NONMEASURABLE!

A physical distance should satisfy two conditions: i) it is geometrically meaningful (possibly geodesic length), ii) it is measurable in practice. In cosmology the conditions *cannot both* hold. Therefore the commonly accepted concept is to introduce a measurable "pseudo-distance" (this name is not used for obvious reasons). Two such notions are used in practice: *luminosity distance* d_L and *area (angular diameter) distance* d_A .

3.1 Luminosity distance

Let L be the (known) absolute luminosity (energy emitted per 1 s) of G and let l be the apparent luminosity (energy received by the unit area of the telescope mirror per 1 s). Assuming that G emits isotropically and its distance in Euclidean space \mathbf{E}^3 is d_L , one gets

$$l = \frac{L}{4\pi d_L^2}.$$

In Robertson–Walker cosmology one *defines* the luminosity distance by *assuming* that the relationship of l to L is the same as in flat space, then

$$d_L \equiv \left(\frac{L}{4\pi l} \right)^{1/2}.$$

This quantity may be expressed in terms of Robertson–Walker geometry. Let t_1 be the instant of emission of light by G and t_0 — the observation of G on Earth, $t_0 > t_1$. If r_G is the radial coordinate of G and $r = 0$ for Earth, it may be shown that (Weinberg, 1972)

$$d_L = \frac{R^2(t_0)}{R(t_1)} f_k(r_G).$$

3.2 Area distance

One makes an analogous assumption: in the cosmological spacetime the relationship between the linear galaxy diameter D , the angle by which the diameter is observed δ and the distance d_A is as in \mathbf{E}^3 , $D = d_A \delta$. In the spacetime it is derived that (Weinberg, 1972)

$$d_A = R(t_1) f_k(r_G).$$

I emphasize that both d_L and d_A have no deeper meaning in the spacetime geometry (Euclidean geometry, where they play a fundamental role, is very distinct from the Lorentzian geometry) and they work as merely *qualitative* indicators of distance. We have nothing better. Using the notion of the redshift z (related to the present instant t_0) and its relationship to the scale factor, $R(t_0)/R(t_1) = z + 1$, one finds the ratio of the two pseudo-distances,

$$\frac{d_L}{d_A} = (z + 1)^2.$$

This relationship is important since it provides a test of isotropy and homogeneity of space (of Robertson–Walker geometry). On the other hand it shows that both quantities are not quantitative indicators of the (unmeasurable) geometric distance. The farthest observed galaxies have $z \cong 7$ which implies $d_L \cong 64 d_A$. Out of these two d_L is better: if R grows monotonically, then d_L grows monotonically with increase of r_G whereas d_A may have maximum for growing r_G and decreasing $R(t_1)$. Nevertheless the statement that a galaxy is at $d_L = 5Gpc$ does not mean much more than that it is very distant. The redshift z is also a distance indicator.

4 What is velocity of a galaxy relative to the Earth?

It is unclear how to define it. In special and general relativity the velocity is a secondary notion. One defines a particle's four-velocity vector u^α as the tangent vector

to its worldline, $u^\alpha = dx^\alpha/ds$ and this quantity has an absolute geometrical sense. By the velocity we traditionally mean the rate of change of particle's position relative to some reference frame. This concept (and measurement) is based on a coincidence. Let an observer (defining its reference frame) has 4-velocity U^α and coincides with the particle with u^α . Then the scalar product of the two 4-velocities makes sense and the particle's velocity v relative to the observer is the positive solution of

$$U^\alpha u_\alpha = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}.$$

We always measure the relative velocity of nearby objects, because only if they are very close to us, the velocity is conceptually and operationally reliable. This is impossible in cosmology. One may generalize the notion by defining it as the rate of change of a distance to a (remote) object. The change of geodesic distance, $v_g = \frac{d}{dt} d_g$, is nonmeasurable as the distance is. The change of the proper distance, $v_p = \frac{d}{dt} d_p = \frac{d}{dt}(Rr) = H(t)d_p$, where $H(t)$ is the Hubble parameter, is equally nonmeasurable for distant objects. Yet for nearby galaxies one can first make an approximation, $d_p \cong d_L$, and then a trick: since the gravitational redshift is observationally indistinguishable from the relativistic Doppler redshift (due to a relative velocity in flat spacetime), one may identify them. For small relative velocities the ratio of emitted frequency ω to the observed one ω_0 , is $\omega/\omega_0 = 1 + z \cong 1 + v/c$ or $v \cong cz$. Finally identifying v_p with v one gets the *linear* Lemaître-Hubble law for the recent recession velocities of galaxies,

$$v \cong cz = H_0 d_L.$$

I stress that this law is (approximately) valid only for $z \ll 1$. Yet the velocities v_g and v_p which are not velocities of a physical motion and arise due to the expansion of the spacetime, are not subject to limitations of special relativity and may be arbitrarily large.

5 What is the Universe?

Up to now I have been discussing various aspects of the Universe without stating what it is. A principal issue arising here is *whether there is only one physical Universe*. I mention this because according to string/M theory fashionable in some circles of theoretical physicists, there is not one Universe, but it is replaced by the *multiverse* — a collection (almost infinite) of distinct “universes” governed by distinct physical laws. I do not wish to discuss this concept; I belong to a large group of physicists claiming that dealing with the multiverse is metaphysics in a mathematical disguise. I presume that the Universe is one. Attempts to define it give rise to a number of definitions.

1. “The largest set of objects and events to which physical laws can be applied consistently and successfully.” (Bondi, 1952)
2. “In cosmology we try to investigate *the world as a whole* and not to restrict our interest to closed subsystems (Earth, Solar system, the Galaxy).” (Sextl & Urbantke, 1983)
3. “The Universe is all that there is.” (Ellis, 2014)

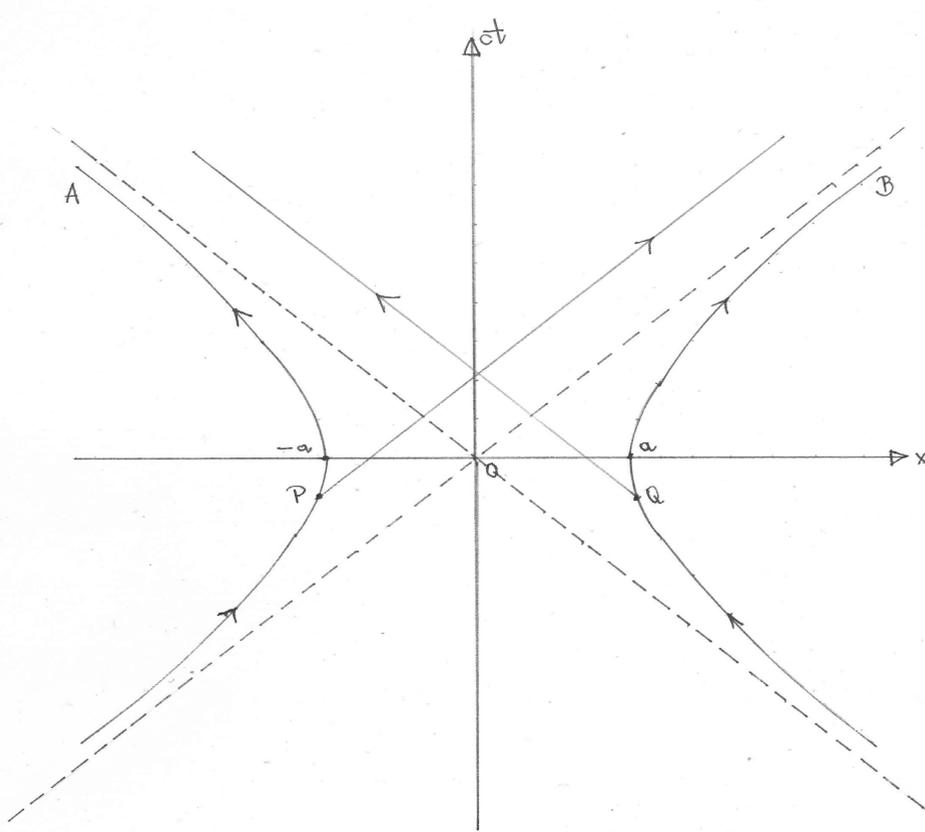


Fig. 2: Three observers in Minkowski spacetime. An inertial observer whose worldline coincides with the entire time axis of an inertial reference frame, sees all events in the universe. The observers A and B moving at a constant acceleration, follow hyperbolic worldlines and emit light signals towards each other. Due to the acceleration they are able to escape from light and the signal sent by A at P will never catch B and the signal sent by B at Q will not reach A. Each of them can see only half of the universe and the division is determined by the light cone emanating from the origin.

These definitions, though intuitively clear, raise doubts. What is “all”? For whom? An example from special relativity: an inertial observer and a uniformly accelerated observer (in hyperbolic motion), Fig. 2. Take the flat spacetime and an inertial frame and consider three observers in this frame. One observer, inertial, stays at rest in this frame and their worldline coincides with the time axis. If this observer exists and makes observations from minus to plus infinity in time, they will be able to register all events occurring in the world and their collection of observations corresponds to the entire Universe according to Ellis. However, two other observers, A and B, move with a constant acceleration and their worldlines form two branches of the hyperbola $c^2t^2 - x^2 = -a^2$ for some $a > 0$. When time $t \rightarrow \pm\infty$, their velocities tend to $\pm c$ and their worldlines asymptotically approach the light cone with the vertex at $ct = x = 0$. A and B emit light signals towards the other observer. The signal emitted by A at P and signal emitted by B at Q move on null lines parallel to the light cone. This

means that the signals will never reach the other observer. At $t = 0$ A and B are momentarily at rest at the distance $2a$, which may be arbitrarily small, nevertheless they will never see each other and one of them does not physically exist for the other. (One can escape from the light, but it requires a constant acceleration.) Each of the two sees (physically registers) only half of the Universe. The definition is observer dependent and in a curved spacetime it is unclear which observer is the correct one and whether an observer capable of seeing the whole spacetime exists at all.

The non-existence of the required observer is clear in a spacetime containing a black hole. There appears the problem of what exists (“there is”). Do objects under the event horizon of a black hole exist for an external observer?

In all these definitions there appears (explicitly or implicitly) the notion of “everything”. This raises a fundamental doubt; in pure mathematics, in set theory, there is Cantor’s antinomy: the concept of a “set of all sets” is a contradictory notion. Are these definitions logically consistent? Due to their vagueness it is difficult to establish this.

There is also a mathematical definition implicitly contained in the famous monograph *The Large Scale Structure of the Space–Time*: “The Universe is the maximal analytic extension of some exact solution of Einstein field equations belonging to the class of cosmological solutions” (Hawking & Ellis, 1973). This definition has the following consequences.

1. There are domains of spacetime which are fundamentally inaccessible to our observations: under the horizons of a black hole, the other exterior domains (which exist in Kerr, Reissner–Nordström and other black-hole spacetimes).
2. The solutions are structurally unstable if the angular momentum $J \rightarrow 0$ and/or the electric charge $Q \rightarrow 0$. This indicates that analyticity is unrealistic.
3. This definition is based on presently accepted physical theory (general relativity).

Without entering in details of these consequences I make a general remark going far beyond this definition: in cosmology we *cannot* use and we *never* use exact solutions. The physical contents of the Universe are too rich and diverse to allow for an analytic solution.

I conclude that the Universe is *not* a well defined physical object. Most cosmologists do not worry about it and in *everyday practice* cosmology goes well without it. This is so because whatever the Universe is, as a whole it is not accessible empirically to us. Only a part of the world, depending on its properties and our observational facilities, is accessible. This “observable Universe” is called *Metagalaxy*, this term appeared around 1930.

6 Uniqueness of the Universe

The Universe is unique and only a part of it, the Metagalaxy, is observable: all information about the world comes from it and all cosmological theories are tested on it. Yet the one Universe is described (in principle) by infinity of mathematical models in general relativity what means that the theory is over-excessive with respect to the world, causing various problems — some of them were recognized quite recently.

From the epistemological viewpoint it would be most adequate to have a one-to-one correspondence: one world — one theory with only one solution (state of the world) —

that is observed. Though highly desirable, this is clearly impossible, this would be in a direct contradiction to the whole evolution of human cognition of the world around us, to the science. In all sciences it is assumed at the outset that there is unlimited number of objects under investigation and accordingly to it any scientific theory reveals a clear distinction between general laws with (practically) infinite number of possible states and initial (boundary) conditions which are determined either by observations or actively by an experiment and which establish which state occurs in reality. This distinction shows that a physical theory comprises components of two kinds: laws of physics — necessity, whereas initial conditions are contingent (chance).

One Universe cancels any distinction between laws and initial conditions.

We must describe the unique Universe employing physical theories providing the over-abundance of cosmological models. Whilst dealing with them we must keep in mind four consequences of the uniqueness. I present them following (Ellis, 2007).

Thesis 1

The Universe itself cannot be subjected to physical experimentation.

We can only passively observe it along our past light cone. This sounds trivial, but its consequence is not. Since we cannot re-run the Universe with altered initial conditions, this means that the actual initial conditions for it are *absolute* and *unchangeable*. In spite of it we believe (?) that they are contingent.

Thesis 2

The Universe cannot be observationally compared with other universes.

There is NO statistical ensemble of physically existing other universes, hence we cannot establish if actual evolution of the Universe is “typical”. The issue of “typicality” concerns the important part of present-day cosmology, the theory of structure formation.

Thesis 3

The concept of “Laws of Physics” that only apply to the Universe as a whole is questionable.

This thesis has two aspects. Firstly, it expresses in different terms the statement made at the beginning of this section, that there is no theory with one state adapted to the Universe. Secondly, it concerns the origin of the initial conditions for the world. General relativity, supported by other branches of physics, provides a huge number of models of the world’s evolution and we wish to know what determined the initial conditions choosing the model that has been realized. I emphasize that in physics we always assume that initial conditions for evolution of the given object are contingent. Actually they are not truly contingent, because they are determined, via other laws of physics, by the object’s environment; we assume that they are contingent since we are interested in the evolution of the object rather than in the evolution of the surrounding and usually we know less about its properties. If we wish we can investigate the evolution of the surrounding with initial conditions determined by its farther surroundings (and these are contingent for us). The Universe has no surrounding imposing some initial conditions for its evolution. Are these conditions genuinely contingent? It would be against the well empirically confirmed thesis that the nature is deterministic. To avoid this conclusion there has been introduced the concept of “laws determining the initial conditions for the Universe”. Thesis 3 makes this concept questionable. Since for this unique object there is no discrimination between laws and initial conditions, the concept loses any meaning. We are strongly accustomed to the notion of initial conditions, but in the case of the Universe the notion does not work properly and we have trouble replacing it by something

more adequate.

I emphasize that genuine physical laws apply to the constituents of the Universe, atoms, stars, galaxies.

Thesis 4

The concept of probability cannot be applied to the unique Universe. Observationally we cannot check if our Universe is “typical” — we see no other worlds. Furthermore, theoretically we cannot say whether our Universe is less or more probable. The latter is not obvious. On the contrary, for several decades after the advent of physical cosmology (the discovery of the relic background radiation in 1965) it was believed that general relativity shows that the world is almost improbable and this required an explanation. A common standpoint was the following: compare the Universe with the mathematical statistical ensemble of universes (cosmological solutions to Einstein equations). A “typical” universe is represented by a generic cosmological solution. (A mathematical definition of a generic solution to nonlinear differential equations is not clear, but this difficulty does not matter. Experts do agree on which classes of solutions of the Einstein field equations are generic and which are special ones.) Robertson–Walker geometry is the most specialized (highest symmetry) cosmological solution, hence it is extremely exceptional. Why is our Universe so improbable? This riddle has been considered as very important and many studies have attempted to explain it.

Now the viewpoint (though not commonly shared) is that comparing the real Universe with the mathematical ensemble makes no sense. One should not ask if the Universe is typical or not. Being unique, the world is beyond this alternative. Instead of it physics *should* explain why the Universe is as it is. At present this aim is beyond our reach.

The conclusion is clear: the philosophical idea that the Universe is *generic* (*typical*, *probable*), or the opposite, cannot be formulated in physics and is unverifiable.

7 Fine tunings

The uniqueness has moreover one far-reaching consequence which we do not understand well: this is the presence of various *fine tunings*. These have been known for many years, have caused a lot of astonishment which have been expressed in the statement that our Universe is very improbable. “Probable or not” — this requires a statistical ensemble of objects and now we know that the ensemble does not exist and we cannot argue in terms of probability. Nevertheless some fine tunings exist and we cannot ignore them. If values of two independent physical quantities are fine tuned, this requires some explanation because it is improbable that it has occurred by chance.

Assuming that there is a deterministic explanation of why physical spacetime has the Robertson–Walker geometry with flat spaces, one encounters in cosmology two main fine tunings.

1. The apparent accelerated cosmic expansion is usually accounted for in terms of some cosmic fluid with large negative pressure. The fluid is interpreted either as the cosmological constant or as a dynamical dark energy. In both the cases the fluid energy density evolves independently of matter energy density (including dark matter) evolution, hence the two energy densities should differ by many orders of magnitude. Yet they are fine tuned:

$$\rho_{\text{DE}} \cong (2 - 3)\rho_{\text{DM}}.$$

2. When the spacetime geometry and the rate of the cosmic expansion find their explanation, there still will be the riddle of the matter composition of the Universe: why the Universe admits existence of life (is “biophilic”). A standard explanation based on Weak Anthropic Principle has been:
 - a) there is a statistical ensemble of universes with different values of ρ_{DE} , ρ_{DM} and nuclear constants,
 - b) we could only appear in a universe which admits us, though this world is less probable (a common sense argument).

Since the ensemble does not exist, the fine tunings need a different explanation. An explanation should exist since no physical law excludes fine tunings. The biophilic feature of the Universe is based on nuclear forces determining properties of various nuclei. I shall now present them briefly.

8 Nuclear forces and life in the Universe

The physical parameters determining the strength of the nuclear forces (strong interactions) are crucial for the material structure of the Universe and life. In this aspect three facts from nuclear physics are essential:

- helium has only 2 stable isotopes, ^3He and ^4He ,
- isotopes ^2He (“diproton”) and ^5He are extremely short living,
- there are no long-living nuclei with atomic mass $A = 5$ and 8.

These facts are crucial both for the primordial nucleosynthesis in the early Universe and for the existence of stars in “mature” Universe.

1. In the early Universe protons and neutrons combined into ^4He nuclei. This process was delayed in time since at an intermediate stage of this reaction deuterium nuclei must be formed and these are weakly bound (as compared with helium) and are easily disintegrated by high energy photons. Only when the temperature of the primordial plasma was sufficiently low and the number of energetic photons sufficiently decreased, could deuterium nuclei survive and produce ^4He . By that time many neutrons decayed and their number was seven times smaller than the number of protons. All these neutrons entered the ^4He nuclei and there were plenty of free protons ready to participate in other reactions. Yet the “mass gap” $A = 5$ and 8 in the sequence of (almost) stable nuclei precluded forming further nuclei: both $^4\text{He} + p$ and $^4\text{He} + ^4\text{He}$ reactions do not produce stable nuclei. The primordial nucleosynthesis (practically) stopped at helium. Most free protons survived from the early Universe, recombined into free hydrogen and later could form stars.

An alternative universe with stronger nuclear forces would comprise different material ingredients. Had the stable diproton and nuclei with $A = 5$ or 8 existed, then all the chemical elements would have been synthesized. One million years after the Big Bang such a universe would be filled with an atomic gas of heavy elements with a small amount of hydrogen (and helium). Today that world would consist of rocky planets and dust without nuclear fuel, in consequence without stars — dark and very cool (3 K) and clearly without any life (no sources

of energy). Sufficient for this radical change would be only tiny changes of the physical constants determining nuclear forces.

2. In a typical star (like the Sun) hydrogen is burnt into helium. There are no free neutrons and the p–p cycle is different from helium synthesis in the early Universe. Nevertheless it must go through deuterium production and the latter is synthesized from two protons forming first the diproton, then inverse beta decay must occur in this nucleus, converting one proton into neutron. Since the diproton lifetime is extremely short, this combined reaction occurs very seldom (its probability is so little, that it will never be observed in laboratory). The helium production goes very slowly and the star burns steadily for 10 billion years.

Again consider an alternative world with stronger nuclear forces and suppose that a large fraction of hydrogen survived from the early epoch (though it seems rather unlikely). Let a cloud of hydrogen collapse and heat up to ignite thermonuclear reactions. Had the stable nucleus ${}^2\text{He}$ existed, then all the protons would have combined in diprotons in few seconds and then into heavier nuclei. The total nuclear energy the star in our world emits in 10 billion years, would be released there in few seconds or at most in several dozens of minutes in an explosion more powerful than supernova explosions. The star would be destroyed together with all its planets.

9 Global evolution and spacetime singularities

Up to now I have been discussing cosmological issues concerning the present-day Universe and the world at an early epoch (assuming that there was an era when the whole of space was filled with a hot plasma of elementary particles). Now I come to the issues arising from the theoretically-recovered evolution of the Universe. I presume that the reader is familiar with the dynamical foundations of the FLRW cosmology. I stress that the matter content of these models cannot be arbitrary matter since the symmetries of Robertson–Walker spacetime must also be the symmetries of the matter energy-momentum tensor and this requirement is satisfied only by a perfect fluid. By a perfect fluid we mean any matter with the energy-momentum tensor (including the cosmological constant)

$$T_{\mu\nu} = (c^2\rho + p)u_\mu u_\nu - pg_{\mu\nu},$$

where $c^2\rho$ is interpreted as energy density and p as pressure (I use the units with $c \equiv 1$), and u^μ is the 4-velocity of comoving droplets of cosmic fluid (of fundamental observers). Usually it is assumed (and it is a good approximation) that $p = p(\rho)$ — the *barotropic* equation of state (EOS). The Einstein field equations are reduced in this spacetime to the Friedmann equation and deceleration equation, whereas the equations of relativistic hydrodynamics, $\nabla_\nu T^{\mu\nu} = 0$, provide the equation of motion for the fluid (all these are shown below).

To get a deeper understanding of what kind of evolution general relativity predicts for our Universe, it is better to see it in a wider perspective. In a generic spacetime the matter source may be arbitrary and to make qualitative predictions it is necessary to make general assumptions about the source. Classical matter should satisfy physically reasonable conditions holding for all known forms of matter. These are *energy conditions*. Here we need the *Weak Energy Condition* (WEC) and the *Strong Energy Condition* (SEC) and in the case of a perfect fluid (this form of matter is relevant

in any cosmology, not only in Robertson–Walker geometry) they take the form of inequalities,

WEC: $\rho \geq 0$,

SEC: $\rho + p \geq 0$ and $\rho + 3p \geq 0$.

SEC is broken in a perfect fluid if at $\rho = 1g/cm^3$ the pressure is $p < -10^{15}$ atm. SEC is also broken if the cosmological constant $\Lambda > 0$ and is driving the evolution (the energy density associated with Λ is larger than (luminous and dark) matter energy density).

It was shown in the late 1960s by S. Hawking and R. Penrose that the energy conditions determine *global evolution* of the spacetime. They proved around 1970 the *generic singularity theorems* in the framework of general relativity:

under reasonable physical conditions singularities develop in spacetime.

What is a singularity?

There is no complete and fully satisfactory definition. Singularity theorems are expressed in terms of geodesic incompleteness. In a spacetime free of singularities each timelike and null geodesic may be infinitely extended to the future and past — these geodesics are *complete*. If a timelike or null geodesic is stopped at a point and cannot be extended farther — it met a singularity. Spacetime has an “edge” — a particle or photon history (worldline) has a beginning or an end.

A singularity is NOT a point, it is a *singular boundary of the spacetime* and the latter is a singular 3-dimensional hypersurface — a 3-dimensional set of points.

Example: the initial singularity in Robertson–Walker geometry is the set of points (r, θ, ϕ) at $t = 0$ and this hypersurface is *metrically* contracted to a point — all distances on it are equal zero. Other well known singularities are the final singularity of the closed ($k = +1$) Friedmann model and the Schwarzschild singularity at $r = 0$. These three singularities are *spacelike*. A hypersurface is spacelike if at each point its orthogonal vector n^α is *timelike*, $n^\alpha n_\alpha > 0$.

Singularities are not due to high symmetry, as was believed for almost fifty years, they are common in physically reasonable spacetimes. On the contrary, regular spacetimes (free of singularities) are exceptional, one of few examples are plane gravitational waves. Singularities (geodesic incompleteness) are frequently also curvature singularities. The Riemann curvature tensor $R_{\alpha\beta\mu\nu}$ has 14 independent scalar algebraic invariants and the simplest of them are $R^\mu{}_\mu$, $R_{\alpha\beta}R^{\alpha\beta}$ and $R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$. A *curvature singularity* exists if some of the scalars diverge to infinity. For example the spacetime of a Schwarzschild black hole satisfies the vacuum Einstein equations $R_{\alpha\beta} = 0$ and the simplest nonvanishing curvature invariant is

$$R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} = \frac{48M^2}{r^6} \rightarrow \infty \quad \text{for } r \rightarrow 0.$$

For known cosmological spacetimes singularity is a curvature singularity.

10 The initial singularity of FLRW models

In the case of Robertson–Walker spacetime the Hawking–Penrose singularity theorem reads: if $\Lambda \leq 0$, SEC holds ($\rho + p \geq 0$ and $\rho + 3p \geq 0$) and $H(t) = \dot{R}/R > 0$, then a curvature singularity occurs in a finite past of any point.

The singularity is the beginning of time ($t = 0$) and by *early Universe* we mean times close to $t = 0$. Physical considerations show that in the early Universe all matter is ultrarelativistic with the equation of state as for the photon gas, $\rho = 3p$,

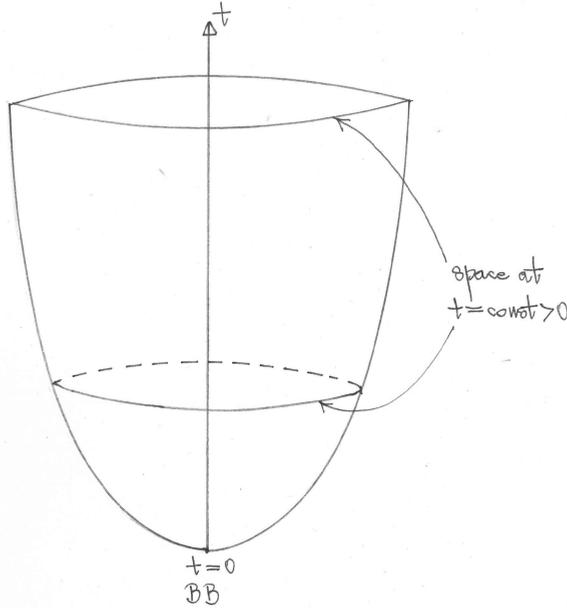


Fig. 3: A closed Friedmann universe emerges from the initial Big Bang singularity $t = 0$. For $t < 0$ there is no spacetime. At each instant $t > 0$ space forms a three-sphere with increasing radius. The cosmic scale factor R and the radius of space grow fastest soon after the Big Bang.

then applying the propagation equations one gets $R^\mu{}_\mu = -8\pi G(\rho - 3p) = 0$ and $R_{\alpha\beta}R^{\alpha\beta} = (8\pi G)^2(\rho^2 + 3p^2)$. The singularity means that $R(t) \rightarrow 0$ for $t \rightarrow 0$ and all spatial volumes (proportional to R^3) tend to zero. In consequence $\rho \rightarrow +\infty$ and $R_{\alpha\beta}R^{\alpha\beta} \rightarrow +\infty$ — a curvature singularity.

The initial singularity in FLRW models may be derived by elementary calculations without resorting to the singularity theorem. We assume $\Lambda = 0$, then the propagation equations are

$$\text{Friedmann eq.} \quad \dot{R}^2 + k = \frac{8\pi G}{3}\rho R^2,$$

$$\text{deceleration eq.} \quad \ddot{R} = -\frac{4\pi G}{3}(\rho + 3p)R$$

$$\text{and eq. of motion of the fluid} \quad \dot{\rho} = -\frac{3\dot{R}}{R}(\rho + p).$$

We assume that SEC holds and $\dot{R} > 0$, then $\ddot{R} < 0$ and $\dot{\rho} < 0$. First one notices that the Friedmann and deceleration equations do not allow static solutions, $\dot{R} \neq 0$, if $\rho > 0$. The Universe must evolve. Observations show that $\dot{R}(t_0) > 0$, hence R is growing about t_0 .

Assume that $\dot{R} > 0$ for $t_m < t \leq t_0$ and $\dot{R} < 0$ for $t < t_m$, or R has a minimum at

$t = t_m$. At the minimum:

$$0 < \ddot{R}(t_m) = -\frac{4\pi G}{3}(\rho + 3p)R \Big|_{t_m} \Rightarrow \rho + 3p < 0,$$

and SEC is broken. It must be $\dot{R}(t) > 0$ for $t \leq t_0$ and $\ddot{R}(t) < 0 \Rightarrow$ for $t = \tau$ in a finite past there is $R(\tau) = 0$. One sets $\tau \equiv 0$ — the *origin* of time, $R(0) = 0$. If $R(0) = 0$, then all spatial distances between different points are 0,

$$d\sigma^2 \equiv -ds^2 \Big|_{t=0} = R^2(0)[dr^2 + f_k^2(r)d\Omega^2] = 0,$$

the space at $t = 0$ is contracted *metrically* to *one* point (though it still remains a 3-dimensional set of points). For $t > 0$ the space *expands* and this implies that $t = 0$ is the origin of space and actually *the origin of the spacetime*. The whole spacetime emerges from the initial singularity. The British astronomer, Fred Hoyle, was an opponent of the concept of emergence of the world from the singularity and around 1948 mockingly called it *Big Bang*. Nowadays this is a respected scientific term.

Clearly the curvature singularity implies geodesic incompleteness. Time and space cannot be extended to $t < 0$ — there is no time, no space, nothing. In Fig. 3 we depict a closed ($k = +1$) Friedmann spacetime.

The pressure uniformly tends to infinity when the singularity is approached. If one makes the inversion of time, then the expansion from the Big Bang is turned into a collapse to the singularity. I emphasize that a uniform pressure (no matter how large) cannot prevent the collapse, it is only a sufficiently large pressure gradient that can do so.

Is it possible to avoid the singularity in our world? Theoretically there are three possibilities.

1. $\Lambda > 0$. The relic CMB radiation has $z \approx 1000$, which implies that the expansion from the recombination epoch up to now is $R(t_0)/R(t_{rec}) \approx 1000$. It may be shown that the singularity can be prevented if

$$\rho_\Lambda \equiv \frac{\Lambda}{8\pi G} > (1000)^3 \rho_M(t_0),$$

this value is observationally excluded.

2. SEC is broken by $\rho + 3p < 0$ — requires extreme quantum effects and is unlikely.
3. A different theory of gravity, then the singularity theorem does not hold. Now this approach is fashionable, but all data clearly show that GR is the best.

11 The early Universe

Let us go back in time to the Big Bang (BB) singularity: $t \rightarrow 0^+$, all volumes $\rightarrow 0$, $\rho \rightarrow \infty$, $p \rightarrow \infty$, matter is *adiabatically* squeezed and gets *relativistic*, then its temperature $T \rightarrow +\infty$ and $p \approx \rho/3$, in consequence all matter takes the form of a structureless plasma of elementary particles — like a photon gas with the Planck spectrum.

Soon after the BB (moving forward in time) there are extreme ρ , T , particle energies — physics beyond the Standard Particle Model. For lower ρ and particle energies (for later times) the Standard Model holds, plasma is in *thermal equilibrium*

— the simplest physical system and this is the Early Universe (EU). Due to its simplicity we believe (with some reservation) that we know the EU better than the present-day Universe. The EU extends from $t \cong 10^{-12}$ s to the recombination epoch, $t_{rec} \cong 5 \cdot 10^5$ years, corresponding to $T_{rec} \cong (6000 - 3000)$ K.

The epoch for $t < 10^{-12}$ s is called the *Very Early Universe* (VEU), it was very short, nevertheless it was very important since it determined most of the main features of the present Universe. The VEU is characterized by extreme particle energies and densities, inaccessible in present and future accelerators — that epoch was ruled by physics experimentally untestable. We see that cosmology inevitably requires physical laws beyond experimental (and observational) reach. For this reason G. Ellis, R. Maartens, M. MacCallum introduced the notion of the *Physics horizon* (Ellis et al., 2012):

a border that separates the experimentally established and verified physics from physical concepts which will never be tested, especially in extremely high energy processes. One can concisely say that the physics horizon limits our knowledge of physical laws governing the VEU. One can only speculate on what occurred in Very Early Universe. Now I briefly comment on these speculations.

12 Quantum Gravity era

The history of VEU and EU is traditionally divided into a number of periods called *eras*. It is conjectured that the first era after the BB is the Quantum Gravity (QG) Era. Why the QG era? The idea of quantum gravity is a great myth of modern physics and as such deserves some attention independently of its relevance for cosmology. It is based on two postulates (being actually conjectures).

1. Planck units.

In the spacetime of dimension $\text{dim}=3+1$ all dimensional physical quantities are built of units of mass M , length L and time T . Yet the units L , M , T are arbitrary and have purely historical origins: meter, kilogram, second. A more physical system of units is based on the 3 fundamental constants of the nature: \hbar — since all matter is of quantum nature, c — all matter is relativistic and G — all matter gravitates. Each physical quantity Q may be expressed dimensionally as a product

$$[Q] = [\hbar^x c^y G^z] = (\text{g cm}^2 \text{s}^{-1})^x (\text{cm s}^{-1})^y (\text{cm}^3 \text{g}^{-1} \text{s}^{-2})^z,$$

where x , y and z are integers or half integers. In this way the dimension of length is $[L] = [\hbar^x c^y G^z] = \text{cm}$ implying $x = z = 1/2$ and $y = -3/2$. One gets *Planck units* (Max Planck 1900):

$$\begin{aligned} l_P &= \sqrt{\frac{\hbar G}{c^3}} = 1,6 \cdot 10^{-33} \text{cm}, \\ t_P &= \frac{l_P}{c} = \sqrt{\frac{\hbar G}{c^5}} = 5,4 \cdot 10^{-44} \text{s}, \\ m_P &= \sqrt{\frac{\hbar c}{G}} = 2,2 \cdot 10^{-5} \text{g}, \\ E_P &= m_P c^2 = 1,2 \cdot 10^{19} \text{GeV}, \\ T_P &= \frac{E_P}{k_B} = 1,4 \cdot 10^{32} \text{K}, \dots \end{aligned}$$

This scale is either *very large* or *very small* — it does not fit known fundamental processes. For this reason the Planck units were ignored for seventy years. In late

1970s it was recognized that \hbar , c and G should together be relevant for *quantum gravity* effects.

Conjecture: *the Planck units determine the scale at which quantum effects dominate in gravitational interactions.*

2. Experimentally it is known that all matter and all interaction fields (except gravitation) are quantum. Observationally known gravitational effects are purely classical (including recently discovered gravitational waves). Yet gravitation is the only universal interaction and this fact strongly suggests that also the gravitational field should have quantum nature. Since all observed gravitational fields are weak and we see no quantum effects in them, quantum gravity should only appear in extremely strong and variable fields near the Big Bang and black hole singularities. If quantum gravity effects dominate in the first era after the BB, we *need* a theory of quantum gravity.

After seventy years of intensive work of many most eminent physicists the real outcome is very modest. There is nothing but difficulties and obstacles of all possible kinds, no hints from experiment or observations. To formulate a quantum theory of gravity is definitely the most ambitious and difficult intellectual task in the whole history of mankind.

For cosmology it is relevant that close to the initial singularity some quantum gravity effects should be dominant, yet nothing more can be stated at present. It is unknown whether the classical picture of the spacetime singularity will be replaced by something radically disparate.

The conclusion is both clear and pessimistic: *we know nothing at present about the quantum gravity era and we expect that if the theory is created in future, we will never test it.*

13 The next era: Grand Unification Era

In the QG Era there is no spacetime (it is replaced by something quantum) and thus no time, therefore frequently appearing suggestions that it lasted for period of order of Planck time t_P are vague and rather misleading. It is conjectured that mysterious quantum processes finally resulted in formation of the classical spacetime and time appeared. The QG Era terminated and the world entered the next era. Now spacetime is described by Einstein's GR. Particle energies and interactions are beyond the Standard Model and it is conjectured that some Grand Unified Theories (which gave their name to this era), still remaining in an embryonic form and unconfirmed experimentally, were driving all physical processes in matter. These theories are also hidden under the physics horizon and probably will never be reliably verified.

It is conjectured that in this era two processes which are crucial for cosmology occurred: first inflation and later baryogenesis. Inflationary evolution of the Universe is a vast subject and deserves separate lectures. I will not discuss it here, though it is one of the two leading myths of modern cosmology. I have here only some space to comment on the issue of baryogenesis.

14 Baryogenesis

Why does the Universe consist almost solely of matter? In fact, the Solar system comprises only matter. In cosmic rays we find one antiproton per 10^4 protons. We register no particles produced in the annihilation process $p + \bar{p}$ and infer that, at

present, there are no regions where matter and antimatter collide on macroscopic scale. Yet the Early Universe was filled with dense homogeneous plasma of p and \bar{p} . We know of no processes segregating e^-p from $e^+\bar{p}$ and conclude that at present there are no anti-stars and anti-galaxies.

This is in conflict with fundamental physics. Quantum field theory and the Standard Model prove the *exact symmetry of matter-antimatter*. Consider the VEU after the inflationary epoch: if it were exactly symmetric in particles and antiparticles, then all pairs e^-e^+ and $p\bar{p}$ would later annihilate into photons and today there would be no atoms and no stars, only the 3 K relic radiation. This clearly shows that at beginning of the EU period, at $t \approx 10^{-12}$ s there was an excess of matter over antimatter. How large? One introduces particle number densities, n_B for baryons (actually these are protons and neutrons), $n_{\bar{B}}$ for antibaryons and n_γ for photons (forming a photon gas at thermal equilibrium with baryons and all other particles). From them one constructs a constant parameter

$$\eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \cong \text{const},$$

which is constant throughout the evolution because all number densities evolve as R^{-3} . The primordial nucleosynthesis left some traces which have survived up to now: these are abundances of D, ^3He and ^7Li . From these quantities one infers that $\eta \cong 6 \cdot 10^{-10}$. This means that the relativistic plasma in the Very Early Universe contained approximately one unpaired e^- per 10^9 pairs e^-e^+ and one unpaired p per 10^9 pairs $p\bar{p}$. All the pairs annihilated in early Universe and only a tiny part of its contents, the excess particles have survived and form the present Universe. What did generate the excess after the inflation?

There are various concepts, all highly speculative. Most of them assume that baryogenesis took place in the Grand Unification Era, implying that these processes are hidden behind the physics horizon and are untestable. Some other current concepts locate baryogenesis at the beginning of the early Universe, thus in principle these are verifiable. At present they are in embryonic form.

15 Conclusions

In these lectures I have argued that cosmology, unlike astrophysics, is not a direct union of astronomy and physics and is a very specific science, distinct from all other natural sciences. In cosmology we investigate an object of which we do not know what it is as a whole, and we can now and in the future study only a tiny part of it. The object is one, whereas to describe it we use our physical knowledge adapted to dealing with an unlimited class of such objects; this causes various misconceptions and false directions of research. Cosmology foretells the future of the Universe, but this knowledge is illusory since it cannot be verified. Cosmology describes the present Universe and its present state is strictly determined by its past. Accordingly, cosmology is a mathematized history of the world. It differs from human history in two fundamental features. Firstly, the physical history is far more precise than human history: it not only registers the past facts, it precisely shows the causes of these facts. Secondly, in human history, if one wishes to explain why the present state of a given nation is as it is, it is sufficient to study the history of this nation in the last two or three centuries, since its ancient history (a thousand years or more ago) has a slight influence on the present state. Yet in cosmology the opposite holds, the features of the Universe we observe now were determined by the earliest epochs of its evolution after its origin and

the laws of physics governing these processes are hidden under the *physics horizon* and we can only make untestable speculations about them. Cosmology is in this sense less reliable than other physical sciences.

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