# Self-interaction scalar field as a dark energy mode

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As is well known, the accelerated expansion of the Universe is successfully described by the $\Lambda CDM$ model. However, the nature of the $\Lambda$-term is unclear and the physical interpretation of this constant led us to the famous problem of 120 orders (Sahni & Starobinsky, 2000; Padmanabhan, 2003). Scalar fields are often considered as candidates for the dark energy (see, for example: Matos & Guzman (2001); Matos, Guzman, & Nunez (2000)). Nevertheless, there is no unambiguous criterion for the choice of the field Lagrangian in the scalar field theories.

In the present work, we are assuming that the scalar field source is the trace of the stress-energy tensor of both matter and the field itself. This condition implies that the scalar field equation has the following form

$$(\Box - m^2)\varphi = q(T^\varphi + T^M), \tag{1}$$

the d'Alembertian is $\Box = -\nabla_\mu \nabla_\nu g^{\mu\nu}$, the constants $m$ and $q$ mean the mass and the interaction constant of the scalar field. Equation (1) allows determination of the Lagrangian of the scalar field and matter (Vyblyi & Tarasenko, 2011):

$$L_\varphi + L_M = \frac{1}{2}\left(\frac{\partial_\mu\varphi\partial^\mu\varphi}{1+2q\varphi} - m^2\varphi^2 + C(1+2q\varphi)^2\right)\sqrt{-g} + L_M\left((1+2q\varphi)\, g_{\mu\nu}, Q_M\right), \tag{2}$$

where $Q_M$ stands for matter fields.

If the interaction between the matter and the scalar field is realized with the help of the effective metric

$$f_{\mu\nu} = (1+2q\varphi)g_{\mu\nu} = \Phi g_{\mu\nu}, \tag{3}$$

then the scalar field equation and his stress-energy tensor have the form

$$(\Box - m^2)\varphi = q\left(-\frac{\varphi_{,\mu}\varphi^{,\mu}}{1+2q\varphi} + 2m^2\varphi^2 - 2C(1+2q\varphi)^2 - T^M\right), \tag{4}$$

$$T^{\mu\nu}_\varphi = \frac{\partial^\mu\varphi\partial^\nu\varphi}{\Phi} - \frac{1}{2}g^{\mu\nu}\left(\frac{\partial_\alpha\varphi\partial^\alpha\varphi}{\Phi} - m^2\varphi^2 + C\Phi^2\right). \tag{5}$$

Let us consider the application of the scalar-tensor theory of gravity to cosmology. As a first approximation we consider the field equations without interaction of the scalar field with matter. The cosmological equations, which include the Friedmann equation, the massless scalar field equation and the covariant law of conservation of stress-energy tensor of matter, read:

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3}\left(\frac{1}{2}\frac{\dot\varphi^2}{\Phi} - \frac{1}{2}C\Phi^2 + \epsilon\right), \tag{6}$$

$$\ddot{\varphi} + 3H\dot{\varphi} - \frac{q\dot{\varphi}^2}{\Phi} - 2Cq\Phi^2 = 0, \tag{7}$$

$$\dot{\epsilon} + 3H(\epsilon + p) = 0. \tag{8}$$

Here $H \equiv \dot{a}/a$ is the Hubble parameter. We can obtain the initial condition of scalar field and its first derivative with respect to time from the expressions:

$$\epsilon^{\varphi} = \frac{1}{2}\left(\frac{\dot{\varphi}^2}{\Phi} - C\Phi^2\right), \quad p^{\varphi} = \frac{1}{2}\left(\frac{\dot{\varphi}^2}{\Phi} + C\Phi^2\right). \tag{9}$$

To obtain the value of the constant $C$, we can have to use the so-called slow roll mode. This regime let us get the expression for the dependence of the constant $C$ on the parameter $q$:

$$C = \frac{81\,(1+\omega)^2}{16q^4\,(1-\omega)^3\,\epsilon_{\varphi}}, \tag{10}$$

We have one arbitrary parameter $q$. We will obtain the numerical solution of the cosmological equations when the value of the parameter is $q = 1$. The numerical solution of these equations is shown in the figures.

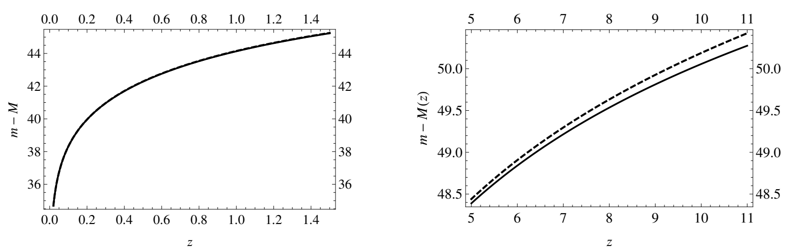


Fig. 1: The luminosity distance $D_{\mathrm{lum}}(z)$ dependence on redshift. The continuous line corresponds to the luminosity distance which follows from our model, and the dashed line corresponds to ΛCDM the luminosity distance. These plots show the luminosity distance curves from $z = 0$ to $z = 1.5$ and $z = 5$ to $z = 11$ redshift scale. The first figure shows that the plots of both models overlap in the redshift scale from $z = 0$ to $z = 1.5$. This means that our model is in good agreement with the supernova observational data such as ΛCDM model. On the other hand, the second figure shows the differences between the models. In our model the supernovae are brighter than in the ΛCDM model at the same large redshift. This means that in our model supernovae are closer than in the ΛCDM model.

## References


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