Between Quantum Mechanics and Cosmology

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We present an outline of our attempts to find relations between quantum mechanics and general relativity based on their mathematical formalisms. We indicate the meaning of set-theoretic forcing and smooth exotic $\mathbb{R}^4$ in physics.

1 Introduction

The mathematical apparatus of quantum mechanics (QM) and general relativity (GR) are both indissolubly connected with the set of real numbers $\mathbb{R}$ (namely, via the formal representations of quantum measurements and spacetime parametrization). The formal structure of $\mathbb{R}$ is studied on the grounds of Zermelo-Fraenkel set theory (ZFC) which is fundamental to almost all of mathematics. Such a real line can vary depending on the choice of a ZFC model (in the sense of model theory). One can describe transition of one model into another via so called *forcing*. According to (Benioff, 1976), implying sufficiently strong definition of randomness, neither the minimal model of ZFC, nor any of its forcing extensions, can grasp the mathematics of quantum mechanics. Abandoning the idea of "static" in-model description we obtain "dynamics" by allowing the changes of actual formal model behind QM formalism that can be characterized in the terms of forcing.

2 The latent meaning of forcing in quantum mechanics

In QM we usually deal with a non-distributive lattice of projections $\mathbb{L}(\mathcal{H})$, a collection of projection operators related to basic yes/no questions about a physical system. Considering some family of pairwise commuting self-adjoint operators one can assign to it some maximal complete Boolean algebra B of projections (Takeuti, 1978). Such B's of projections from $\mathbb{L}(\mathcal{H})$ are atomic. However, the complete algebra of projections made of projections of a family of pairwise commuting self-adjoint operators with absolutely continuous spectra is the measured algebra $\mathfrak{B}$ and is atomless. Such an algebra supports nontrivial (i.e. adding new reals) random forcing $\mathbb{V}^{\mathfrak{B}}/_{[G]}$, where G is a generic ultrafilter on $\mathfrak{B}$. Suppose, that the real numbers which parametrize space come from the quantum realm via continuous measurement ($\sim$position observable van Wesep, 2006). Then this random forcing captures the difference between reals used in QM and the continuum of real parameters describing the spacetime in GR (Klimasara & Król, 2015). Thus we actually face a varying structure of the real line. It can have interesting physical implications, e.g. for the cosmological constant (CC) problem. The discrepancy between CC predictions in particle physics and observations comes from vastly overestimated contributions (to the CC) from zero modes of quantum fields. In our varying-model setup such contributions completely vanish (Bielas, Klimasara, & Król, 2015; Klimasara, Bielas, Król, & Asselmeyer-Maluga, 2016). This follows from direct calculation of zero modes, since we integrate over a set of negligible size (real line before extension by forcing).

3 Cosmological constant and exotic smoothness

We have dealt with zero-point energies in the above sense, but the CC value cannot be zero to fit the experimental data. One can complete the picture by the fact that a realistic value of the CC (and other cosmological parameters) can be obtained by using exotic smooth $\mathbb{R}^4$ (homeomorphic but not diffeomorphic to the standard $\mathbb{R}^4$) as a manifold describing the spacetime in GR (Asselmeyer-Maluga & Król, 2014). Moreover, we can connect both treatments on formal grounds using the Erdős-Sierpiński theorem (Oxtoby, 1971) to switch the measured algebra into a Cohen algebra ($\sim$transition to Cohen forcing). Then we can interpret real numbers from a model and its Cohen extension as absolute subtrees of the binary tree (Cantor space). Such trees are spanning nontrivial Casson handles of smooth exotic 4-manifolds (for more about such manifolds and physics see Asselmeyer-Maluga & Brans, 2007).

4 Summary

Accordingly, varying-ZFC-model-based QM and exotic smooth spacetime constitute complementary descriptions of the micro- and macroscale relation. We propose them as the foundations of a forcing-based cosmological model and we plan to study the consequences of such a description of the Universe in future work.

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