Cosmology from large-scale structure observations: a subjective review

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In these lecture notes we give a brief overview of cosmological inference from the observed large-scale structure (LSS) of the Universe. After a general introduction, we briefly summarize the current status of the standard cosmological model, ΛCDM, and then discuss a few general puzzles related to this otherwise successful model. Next, after a concise presentation of LSS properties, we describe various observational cosmological probes, such as baryon acoustic oscillations, redshift space distortions and weak gravitational lensing. We also provide examples of how the rapidly developing technique of cross-correlations of cosmological datasets is applied. Finally, we briefly mention the promise brought to cosmology by gravitational wave detections.

1 Introduction

The observed distribution of galaxies in the Universe – the *large-scale structure* (LSS) – grew from tiny primordial quantum fluctuations ("seeds" from cosmic inflation) and was shaped by various physical processes acting on the matter distribution from the Big Bang to the present. At large intergalactic and cosmological scales the dominant force is gravity and thus the only relevant process via which the LSS can build up is the relatively well-understood *gravitational instability.* In this process, small density irregularities in the primordial Universe (whose imprint is now observed in the cosmic microwave background, CMB) gradually became larger. With time, overdense regions decouple from the universal expansion to form bound structures, while the initially underdense patches become more and more empty expanding at a super-Hubble rate (*e.g.* Peebles, 1980). In this picture the main building blocks of the LSS are *galaxies*, the formation of which is however difficult to model and study due to various energetic and highly non-linear baryonic physical processes, such as the formation of supermassive black holes (SMBH) and their back-reaction onto the host galaxy via the active galactic nucleus (AGN) phase, star formation and supernova energetic feedback, metal-line gas cooling, chemical gas enrichment, and radiation pressure and reionisation. See *e.g.* Baugh (2006); Mo et al. (2010); Somerville & Davé (2015) for reviews on the physics of galaxy formation.

The LSS is one of the main sources of cosmological information. By measuring and studying its statistical properties we can derive and estimate various fundamental parameters describing the standard cosmological model. Among these parameters are those crucial for describing the global evolution of the Universe: mean non-relativistic *matter density* (both 'baryonic' and dark), usually denoted as Ω_m; the value of the *cosmological constant*, or more generally – the amount of *dark energy*, also parametrised through non-dimensional Ω_Λ; as well as the current *expansion rate* of the Universe, parametrised as the *Hubble constant* H_0 (today) or the more general *Hubble parameter* $H(z)$ where z is the cosmological redshift. The key words in this context are, for instance, the (galaxy) density power spectrum and correlation function, baryon acoustic oscillations, the growth rate of structure etc. Some of these are discussed in more detail in §4.

2 Observational cosmology

2.1 What we already know, or, more precisely: what is most consistent with observations

Decades of cosmological surveys, together with the theoretical framework in which we interpret the observations, supported by numerical simulations, give a generally self-consistent picture of the structure and building blocks of the Universe, summed up in the so-called ΛCDM (Lambda-cold-dark-matter) concordance model. First of all, most of the cosmic matter ($\sim 82\%$ of all matter, according to Planck Collaboration et al. 2016b) is in an 'invisible' – *dark* – form, capable of interacting only via gravitation and possibly weak nuclear forces. Secondly, the universal expansion has been accelerating for the last several billion years (Riess et al., 1998; Perlmutter et al., 1999), which means that dark energy, or some other phenomenon acting in a very similar way, currently dominates the mass-energy balance of the Cosmos (*e.g.* Peebles & Ratra, 2003). Furthermore, space is globally flat, *i.e.* it has vanishingly small curvature and is described by Euclidean geometry on the largest scales, and the total density of all the components of the matter-energy budget add up to the critical density (Planck Collaboration et al., 2016b). Moreover, there is also evidence that the Universe is homogeneous and isotropic on large scales, however the quantitative definition of the scale of homogeneity is still being discussed in the literature (see for instance Pan & Coles 2000; Hogg et al. 2005; Scrimgeour et al. 2012; Alonso et al. 2015; Pandey & Sarkar 2015). Last, but not least, within the observational errorbars we do not currently see any departures from general relativity (GR) in such probes as the growth of structure or gravitational lensing. See, however, the discussion in the following subsection regarding potential pitfalls and shortcomings of such analyses and interpretations.

2.2 Questions... What we don't yet know fully (or at all)

The great successes and ability to explain most of the observations by the ΛCDM model come at the price of accepting that it is mostly phenomenological. This is clearly reflected in the fact that the model introduces some assumptions and entities whose physical nature we still do not understand and which remain untested or undetected. The phenomenological character of the standard cosmological model also makes it not easily susceptible to observational falsification.

Firstly, we do not know what dark matter is, although we know to a large extent what it cannot be. Dark matter is non-baryonic (Steigman, 2007), it is collisionless (*e.g.* Frenk & White 2012), or has at most a very small cross-section for elastic scattering (Harvey et al., 2015; Kim et al., 2016), and it is non-relativistic (*i.e.* with characteristic velocities $v/c \ll 1$), which is the reason why it is dubbed 'cold dark matter' – CDM. Its properties deduced from observations are consistent with it being composed of elementary particles. The most popular candidate consists of the hypothetical lightest super-symmetric particle – the *neutralino* – which should leave an observational imprint through γ-ray radiation produced during self-annihilation[1] (Bergström et al., 1998). Another interesting alternative that has been proposed in the last decade is the *sterile neutrino*, which would produce X-ray radiation through a particle decay process. In addition, the sterile neutrino would have at least a six orders of magnitude smaller rest mass than the neutralino (i.e. in the keV rather than GeV range), hence it would constitute a *Warm* Dark Matter (WDM) candidate. WDM in contrast to CDM has small, yet not negligible, thermal relic velocities, which means that any perturbations smaller than the corresponding free-streaming scale are erased. This can be potentially exploited to design new observational tests to help distinguish between these two scenarios, as the abundance and internal structure of DM haloes around and below dwarf galaxy scales is predicted to be different in these two models (see *e.g.* Boyarsky et al. 2009; Lovell et al. 2012; Bose et al. 2016). In addition, in contrast to WDM models, the CDM model predicts in general the existence of many dark matter haloes not hosting a galaxy (up to the mass scale of dwarf spheroidals), and such a prediction can be tested by using observations of strong-lensing arc distortions (see *e.g.* Li et al. 2016). There are hotly debated claims that the observational signatures of DM of these two different kinds have been actually indirectly detected. For instance, the "gamma-ray excess" observed in the direction of the Galactic centre has been ascribed to annihilation radiation of CDM (Hooper & Goodenough, 2011). On the other hand, the 3.53 keV X-ray line detected in the stacked spectrum of galaxy clusters (Bulbul et al., 2014) and, independently, in the Perseus galaxy cluster and in the Andromeda galaxy (Boyarsky et al., 2014) has been attributed to the decay of a 7 keV sterile neutrino (but see *e.g.* Riemer-Sorensen 2014; Anderson et al. 2014; Malyshev et al. 2014 for different interpretations). The bottom line regarding the existence of DM in the form of particles is, however, that we still lack compelling evidence for it, such as would be provided by a direct detection of a DM particle in an Earth-based laboratory such as the Large Hardon Collider at CERN.

Secondly, we do not know what dark energy is, or more generally, what drives the accelerated expansion of the Universe. Current observations are consistent with a simple cosmological constant Λ, which – if interpreted as a fluid – has an equation of state ($p = w\rho$) such that $w = -1$, which means negative pressure. However, the value of $\Lambda \simeq 1.2 \times 10^{-52}\ \mathrm{m}^{-2}$ as measured from astronomical surveys (Planck Collaboration et al., 2016b) is many orders of magnitude (~ 50) smaller than that which could be predicted from quantum field theory considerations for the energy of the vacuum. This is one among many reasons why various alternative explanations to the accelerated expansion are proposed. Among them is a postulate that the inferred acceleration is an artefact of an overly simplistic cosmological model, for instance due to the so-called *back reaction* (Buchert et al., 2015), although we note that this issue is a matter of a debate (Green & Wald, 2015). Other explanations for the accelerated expansion include modifications to GR on cosmological scales (Clifton et al., 2012) or

[1] The hypothetical neutralino would be a Majorana fermion, which means it would be its own antiparticle and would self-annihilate during random collisions. The amount of annihilation radiation produced in such a process will be proportional to the square of local neutralino DM density.

fluids more exotic than Λ, such as weakly coupled scalar fields like the quintessence (Carroll, 2001). See also Joyce et al. (2016) for a review.

In addition, related to the above discussion is the general question whether GR is an accurate theory on the largest cosmological scales. One of the core assumptions of the ΛCDM model is that GR correctly describes gravity on *all physical scales*, namely from the Solar System and smaller scales (*i.e.* $\lesssim 1$ AU) up to the Hubble radius (*i.e.* the observed cosmic horizon, $\sim$ tens of Gpcs). While GR is very well tested in the former case, as well as in the strong-field regime (Will, 2014), cosmological observations so far only give us very limited information on its validity, as the relevant tests are usually swamped by observational errors or systematics (e.g. Pullen et al. 2016). Thus, using GR to describe the whole observed Universe is an extraordinary and unprecedented extrapolation. This motivated a new field of study that became very active in the past decade and concerns many interesting modifications to GR, collectively dubbed as Modified Gravity theories (MG) These were put forward as explanations alternative to dark energy of the observed late-time accelerated expansion (Carroll et al., 2004; Clifton et al., 2012; Joyce et al., 2015). Here, the observed acceleration would be a manifestation of the breaking of GR on cosmological scales, rather than the signature of an exotic fluid of dark energy type. The MG theories of interest in this context must recover the GR limit on galactic and sub-galactic scales, while potentially allowing for non-negligible modifications of gravity on cluster and intergalactic scales. Conducting stringent tests on cosmological scales of GR alongside its viable modifications has become one of the major goals of 21st-century cosmology. A variety of such observational GR tests have been proposed in the past years (see *e.g.* Jain & Zhang 2008; Jain & Khoury 2010; Hellwing et al. 2014; Koyama 2016). However, the most widely considered and implemented is a parameter that measures the growth rate of structure, quantifying the efficiency of structure formation in an expanding Universe. This statistic can be derived for example by measuring anisotropic clustering of galaxies in so-called redshift space, via *redshift space distortions* (RSD; see §4.2 for more details). Currently the best available data analysed with state-of-the-art RSD models are consistent within the error bars with both GR and also some of its popular modifications (see *e.g.* Samushia et al. 2012; Beutler et al. 2012; de la Torre et al. 2013). Although more and more abundant data, together with better theoretical and computational modelling, provides better and better constraints on GR, there are still plenty of MG models on the market which try to explain the observed accelerated expansion in the cosmological context without spoiling the successes of GR on small scales. Currently planned next-generation spectroscopic surveys such as DESI (Levi et al., 2013), SKA (Abdalla et al., 2015) or Euclid (Laureijs et al., 2011) will provide enormous amounts of new galaxy data which should allow us to significantly reduce the error bars on the growth rate measurements and hence allow for new stringent tests of GR on cosmological scales. We note, however, that for these new surveys the dominant source of errors will be systematics rather than statistics, which will call for very precise modelling of the measured observables in the various MG models to be tested.

Another pillar of the ΛCDM cosmological model still to be comprehensively tested are the so-called *cosmological* and *Copernican principles*. The first one says generally that the Universe is 'the same everywhere'; the second – that we are not a privileged observer (or that we *are* a typical one). Mathematically this first principle boils down to the assumptions that the Universe is homogeneous and isotropic, and on large scales can be described with the Friedman-Lemaître-Robertson-Walker (FLRW) metric. Although there is lots of evidence that the cosmological principle holds (Pan & Coles, 2000; Hogg et al., 2005; Scrimgeour et al., 2012; Alonso et al., 2015; Pandey & Sarkar, 2015, 2016), there do exist related observational puzzles such as anomalies in the CMB (Planck Collaboration et al., 2016c), the origin of which still are not fully understood. The very scale itself of the global isotropy and especially homogeneity is a matter of debate as well (Guzzo, 1997; Wu et al., 1999; Yadav et al., 2010; Maartens, 2011). Furthermore, even if the *cosmological* principle does hold, the *Copernican* one does not have to. Clearly, we as observers on Earth or in the Solar System are *not* in a random place in the Universe, which would more likely be in the middle of a cosmic void (as voids occupy most of the *volume* of the Universe nowadays) and probably not even in a galaxy. How much this fact spoils our inferences about the Universe in general is not fully clear, however it has been recently shown that, for the case of observations concerning peculiar velocities of nearby galaxies, our specific observer location introduces strong systematic effects in the data (Hellwing et al., 2016).

An important ingredient of the standard model is the assumed Gaussian nature of the primordial density fluctuation field. Potential departures from this assumption are described as *non-Gaussianity* and would be signatures of specific models of the mechanism behind the cosmic inflation epoch. Nonetheless, the recent very precise measurements from the Planck mission (Planck Collaboration et al., 2016d) have placed very tight constraints on primordial non-Gaussianity, leaving hardly any room for significant departures from Gaussian initial conditions.

Last but not least, there are many more detailed questions to which we do not have fully satisfactory answers. Usually the main reason why these issues are still not resolved is the very large degree of complications of the so-called 'gastrophysical' (*galaxy+astrophysics*) aspects, which cannot be studied analytically and are also very challenging for simulators, although the latter aspect has greatly improved recently thanks to such efforts as EAGLE (Schaye et al., 2015) or Illustris (Vogelsberger et al., 2014). Among these we could list such problems as the history of galaxy and star formation; build-up of galaxies from primordial gas; effectiveness of stellar and SMBH energy feedback; details of the cosmic reionisation process; galactic magnetic fields; collisions and mergers of galaxies; build-up of galaxy clusters and large-scale voids. . . etc.

3 The large-scale structure of the Universe

According to the standard cosmological paradigm, the LSS has gradually formed out of primordial Gaussian density fluctuations imprinted today in the CMB temperature anisotropy distribution. These primeval perturbations were at a level of $\delta \sim 10^{-5}$ at the moment of decoupling of radiation from matter roughly 380,000 years after the 'moment zero' of the Big Bang. The distribution and properties of these temperature anisotropies have been studied in detail for decades thanks to many successful space-borne and ground-based CMB observatories, with milestones such as COBE (Boggess et al., 1992), BOOMERANG (Crill et al., 2003), WMAP (Bennett et al., 2003) and Planck (Tauber et al., 2010). The well-understood physics of the primordial plasma allows for a simple interpretation of the measurements, leading to a 6-paramter cosmological model which fits observations such as the angular power spectrum of the temperature fluctuations extremely well (Planck Collaboration et al., 2016a).

The primordial density perturbations, which left the imprint in the observed CMB temperature fluctuations, have been evolving through the gravitational instability mechanism: overdense regions collapse under their own gravity to become *galaxies*, *clusters* and *superclusters* of galaxies, while underdense regions expand faster than the background to become *voids* with densities much lower than the average. The system of these structures – organized into a network of interconnected filaments and walls (with clusters and superclusters in the knots), surrounding giant voids – is often called the *cosmic web* (Bond et al., 1996), but it is also reminiscent of foam or sponge. We have evidence that the cosmic web, clearly visible at the present time (e.g. Huchra et al. 2012; Bilicki et al. 2014), was already in place at $z \sim 1$ or more (e.g. Guzzo & The Vipers Team 2013). This observational picture of LSS formation is strongly supported by numerical simulations of the evolution and growth of matter perturbations, where the LSS emerges out of initial conditions at early times consistent with CMB observations.

4 Cosmological inference from large-scale structure

The LSS observations are used in various ways to measure several cosmological parameters and here we will present a handful of probes and techniques. These include baryon acoustic oscillations (§4.1), redshift space distortions (§4.2), weak gravitational lensing (§4.3), cross-correlations (§4.4), peculiar velocities (§4.5) and finally, mentioned as a possible future cosmological probe, gravitational waves (§4.6).

4.1 Baryon acoustic oscillations

Baryon acoustic oscillations (BAOs, Bassett & Hlozek 2010) can be regarded as relics of sound waves (*i.e.* pressure- and gravity-supported oscillations in plasma) propagating through the early Universe in the era when matter was coupled to radiation. These waves "froze" at the moment of decoupling, leaving peaks and dips in the CMB angular power spectrum. The same process also left a characteristic scale in galaxy correlations, often visually depicted as "rings" around galaxy concentrations, although in practice these rings can only be detected statistically and are not visible directly in the LSS. BAOs are seen in the two-point galaxy correlation function (2PCF) as a characteristic peak at a redshift-dependent scale. As this measured angular scale can be compared to the (fixed) well-known one from the CMB at $z \sim 1100$, which can be further rescaled to a linear size at any other redshift, this allows measurement of the "distance to a given redshift" and the Hubble parameter at this redshift, and hence is useful for assessing the time evolution of the expansion rate. This makes the BAOs a powerful standard ruler testing the rate of expansion, and their measurements give one of the pieces of evidence for cosmic acceleration and dark energy. Since the first BAO detections from the 2dFGRS (Cole et al., 2005) and SDSS (Eisenstein et al., 2005), they are now routinely measured from all major redshift surveys, from very low redshifts ($z \sim 0.1$, 6dFGS, Beutler et al. 2011) up to very early epochs of the Universe ($z > 2$ from the Lyman-α forest in SDSS quasar observations, Busca et al. 2013). Nowadays, BAOs are used also for other cosmological constraints, especially when combined with various other probes (Aubourg et al., 2015). Additionally, *angular* (2D) galaxy correlations, relevant for *photometric* redshift surveys (Blake & Bridle, 2005), are also employed.

4.2 Redshift-space distortions

As already mentioned, the LSS forms through gravitational collapse of overdensities. A direct consequence of this is that galaxies do not strictly follow the uniform expansion of the Universe but have additional *peculiar velocities* atop of the Hubble flow, incurred by their motions due to local gravity. These velocities are strongly coupled to the density field and lead to differences between the true distribution of galaxies and that observed with redshift surveys. The measured redshift is composed of the cosmological one resulting from the global isotropic expansion (Hubble law $cz = H_0 r$ for $z \ll 1$) and an additional line-of-sight (LOS) component of the peculiar velocity, i.e. $z_{\rm obs} = (1 + z_{\rm cos})(1 + v_{\rm pec}/c) - 1$, where $z_{\rm obs}$ is the observed redshift, $z_{\rm cos}$ is the redshift due to the uniform expansion of the Universe, and $v_{\rm pec}$ is the LOS component of the three-dimensional peculiar velocity. Hence the notion of the *redshift space* – a coordinate system where the radial distance to a galaxy is denoted by its observed redshift (as opposed to the *real* or *configuration space*). The radial component of the galaxy peculiar velocity vector distorts the distance to a galaxy estimated by assuming that the whole observed redshift is only due to the Hubble recession velocity. This effect gives rise to the so-called *Redshift Space Distortions* (RSD).

The strong coupling of the matter and peculiar velocity fields, and the fact that we observe only the LOS component of $v_{\rm pec}$, result in a specific manifestation of the RSDs: the 2PCF of galaxies measured in redshift space is dependent on the angle with respect to the LOS. Two main effects in such RSDs can be singled out: "squashing" of the 2PCF on large scales ($\gtrsim$ 20 Mpc) – the *Kaiser effect* (Kaiser, 1987) – as well as radial elongation of the PCF, the *fingers of God*, in large overdensities, such as galaxy clusters or groups. The former results from coherent infall on mass concentrations, while the latter effect is due to random motions in virialised regions. The cosmological importance of RSDs is related to the fact that the general shape of the 2PCF, and in particular the amount of squash and elongation, depends on cosmological parameters such as the growth rate of structure. Therefore, by measuring the 2PCF and comparing the observations to various models we can infer the growth rate at various redshifts and check if this growth is consistent with theoretical predictions. In particular, the RSDs are considered a powerful probe of modified gravity models, although we note that fully exploiting them with future surveys, providing much smaller statistical errors than the current ones, will require careful modelling of the 2PCF for those various models.

Together with BAOs, the RSDs are the major probes benefitting from clustering properties of matter, imprinted in the correlation functions measured from spectroscopic redshift surveys. We note that the Fourier space counterpart of the correlation functions – the power spectrum – is also employed for RSD measurements and general cosmological parameter inference.

4.3 Weak gravitational lensing

On their way from astronomical sources to the observer, the paths of photons are distorted by the intervening matter. This effect is called *gravitational lensing* and it is particularly important for LSS studies because it directly probes all types of matter, whether luminous or dark, baryonic or non-baryonic. Of most interest for cosmology is the *weak lensing* (WL) regime, in which observed galaxy shapes undergo tiny distortions (Bartelmann & Schneider, 2001; Refregier, 2003; Hoekstra & Jain, 2008). Galaxy shape changes are correlated on large scales because of density perturbations accumulated between the sources and the observer, and this cumulative effect observed in galaxy imaging is often called the *cosmic shear*.

Weak lensing is arguably the most powerful tool to map the distribution of matter in the Universe, as it gives an unbiased picture of the LSS. Such mapping is done by statistically quantifying distortions of distant source galaxy images. This distortion is often characterised by the lensing convergence κ, which is essentially the projected mass density along the line of sight between the source and the observer. Various statistics of 2D κ maps, such as its power spectrum $C_{\kappa\kappa}(l)$ (Kilbinger, 2015), peak counts (Hamana et al., 2004), or Minkowski functionals (Kratochvil et al., 2012), are used and they often contain complementary information.

The properties of the cosmic shear depend mostly on the amount and clustering properties of matter between the source and the observer. The cosmic shear signal is extracted from correlation functions and power spectra of galaxy shapes (rather than from clustering like in the case of BAO or RSD). Up to now, cosmic shear measurements are, however, not as competitive a cosmological probe as those using clustering measured from redshift surveys. The main reason for this is that, unlike in clustering measurements where only angular positions and redshifts of galaxies are needed, weak lensing measurements require extremely good quality imaging, which in the case of ground-based observations means use of powerful telescopes located in sites with very good observing conditions (very good seeing) – such as Chile or Hawaii. Additionally, to probe large volumes of the Universe and beat the so-called *cosmic variance*[2], weak lensing observations must cover large angular scales. All this has become possible only in the recent years, thanks to such surveys as (already complete) CFHTLenS (Heymans et al., 2012), as well as ongoing DES (The Dark Energy Survey Collaboration, 2005), HSC (Takada, 2010) and KiDS (de Jong et al., 2013). In the coming decade or so, a great leap in cosmic shear measurements is expected from such planned experiments as the LSST (LSST Science Collaboration et al., 2009), and especially Euclid (Laureijs et al., 2011) which will provide the first wide-angle space-borne weak lensing observations.

4.4 Cross-correlations

In this section we will discuss a general *technique* of obtaining cosmological information rather than a specific probe: the approach of *cross-correlations*. It is a generalisation of the auto-correlations discussed in the preceding sections in the context of BAOs, RSDs and cosmic shear. In those applications, specific 2D or 3D correlation functions or power spectra of individual datasets are used – i.e. the data are there *auto-correlated* with themselves. In *cross*-correlations (CCs), the idea is that if two different sets of data probe the same LSS (or its effect on the measurements), then their mutual correlation will give non-zero signal. Cosmological parameters are then inferred from cross power spectra or cross-correlation functions of the two catalogues. CCs benefit from as large sky coverage as possible, because their signal-to-noise usually scales as the square root of the angular coverage. This means that in many applications of this technique, surveys covering the full extragalactic sky would be the most beneficial. This latter requirement is one of the reasons why in many cases the CCs have become practicable only recently, thanks to the ever growing amount of

[2] The fact that cosmological parameters derived from observations of small angular fields will have an unavoidable scatter due to particular realisations of the LSS in these fields.

wide-angle high-quality cosmological data. As far as the technical side is concerned, the CC approach is often executed by using *sky maps* of the given signal, and a standard tool for making such maps is HEALPix (Górski et al., 2005), originally designed mostly for CMB applications but now widely used for many other types of data. CCs are a powerful technique especially if the signal we look for is much lower than other signals in the data. An example here is the integrated Sachs-Wolfe effect (ISW; see below) in the CMB. They also allow mitigation of systematics if these are different between the two cross-correlated surveys, such as instrumental effects. Last but not least, as the CCs are usually done using projected (2D) signals, then in the case of maps derived from galaxy surveys, spectroscopic redshifts are in most cases not required, but rather some amount of information on the galaxy redshift distribution, for instance from photometric redshifts.

4.4.1 Integrated Sachs-Wolfe effect

The above-mentioned ISW effect (more precisely: *late-time ISW*), is a good examples how CCs are used. This effect (Sachs & Wolfe, 1967), as well as its non-linear generalisation, the Rees-Sciama effect (Rees & Sciama, 1968), is a secondary anisotropy of the CMB temperature distribution[3] induced by the fact that photons change their energy on the way from the last-scattering surface to the observer as they pass through potential wells and hills related to the existence of the LSS. The CMB fluctuations generated by the ISW are much smaller than those related to the primary anisotropies, hence this effect cannot be detected from the CMB alone. However, as originally proposed by Crittenden & Turok (1996), the ISW effect can be looked for by cross-correlating CMB and LSS maps. The first detection of the ISW was made once WMAP data became available, from cross-correlating CMB with NVSS radio data and the HEAO1 A1 X-ray measurements (Boughn & Crittenden, 2004). Since then, CCs have become the standard technique to measure ISW, by using WMAP or Planck CMB maps together with various galaxy surveys. As far as the detection of the ISW itself is concerned, such a signal is an independent evidence for dark energy: in a spatially flat Universe such as ours, the ISW is non-null only if the matter density is below the critical value, which means that there must be a non-zero Λ term to close up the matter-energy budget. In principle, studying the ISW in more detail could be also used to constrain dark energy properties; this is however very challenging, as the CC signal is in most cases very low ($\lesssim 4\sigma$).

4.4.2 Gravitational lensing of the CMB

Another example of the usefulness of CCs is the gravitational lensing of the CMB byn the LSS. This effect (Lewis & Challinor, 2006) is another secondary anisotropy of the CMB, which emerges due to the same phenomenon as the above-discussed weak lensing (cosmic shear), only here the source of photons is the last-scattering surface rather than galaxies. The CMB lensing, unlike the ISW, can be detected from CMB observations alone (*e.g.* Planck Collaboration et al., 2014), however cross-correlating its maps with LSS information can be used to probe the growth rate of structure in a way totally independent of such probes as the RSDs. If the CC is additionally done *tomographically*, *i.e.* the LSS maps are generated for particular redshift slices, this in principle allows for deriving the redshift evolution of the growth rate, which can be a powerful probe of GR and MG models. The signal of the CMB lensing vs.

[3] In contrast to the *primary* anisotropies which were generated at the surface of last scattering.

galaxy distribution CCs is currently limited by the limited accuracy of the lensing maps rather than the LSS ones; the former are either noise-dominated (such as the one from Planck) or cover relatively small angular scales (*e.g.* ACT, Sherwin et al. 2012, or SPT, Bleem et al. 2012).

The CC of LSS with CMB lensing has been recently extended to using cosmic shear rather than galaxy catalogues as the tracer of the LSS. The advantage of this approach is that, unlike the observed *galaxy* distribution, which is a *biased* tracer of the underlying density field (*i.e.* $\delta_g = b\delta_m$ in the linear regime where b is the *linear galaxy bias*), the cosmic shear, and more generally weak lensing, probes the LSS in an unbiased way. Thus, cross-correlating CMB lensing with weak lensing maps gives great promise to constrain cosmological parameters without the 'nuisances' related to galaxies. This approach is however relatively new, as both the CMB lensing maps and wide-angle cosmic shear catalogues started being available only in the last ~ 5 years. The first detection of the CC of CMB lensing and galaxy lensing was made only in 2015 (Hand et al., 2015) and as of yet such analyses are rather in their infancy (Kirk et al., 2016). We must wait for more data on both the CMB lensing and cosmic shear before such CCs become competitive cosmological probes. Such data should be brought in by ongoing and planned experiments such as AdvACT or COrE (The COrE Collaboration et al., 2011) for the CMB, and several projects mentioned in §4.3 for the cosmic shear.

4.4.3 Gamma-ray background

Many other maps and surveys are being cross-correlated for various cosmological and astrophysical tests. An interesting example is the CC of galaxy distribution with Fermi-LAT maps of the γ-ray background. In general, γ photons of astrophysical origin are generated in various non-thermal, very energetic processes, for instance in blazars, AGNs, and during star formation. However, several dark matter models in which the DM particle either decays or self-annihilates predict a γ-ray signal from such DM-related events and such a signal should be localized to where structures, such as galaxies and clusters, are[4]. Thus, as we already mentioned in §2.2, one can also look for possible DM evidence (or lack thereof) in this signal. One of the approaches is to use the unresolved γ-ray background (*i.e.* maps of astrophysical γ photons after subtracting the contributions of point sources) as a probe of extragalactic sources[5], and cross-correlate it with galaxy distribution. In 2015 the first detection of such a CC was reported (Xia et al., 2015), however constraints on dark matter from such analyses are so far very weak, partly because the extragalactic γ-ray signal from 'standard' astrophysical processes is expected to dominate over any reasonable DM contribution (Cuoco et al., 2015).

Recently, this approach has been applied also to cosmic shear data as tracers of the LSS rather than source catalogues, for reasons and with expected benefits similar to the case of LSS–CMB lensing CCs (Camera et al., 2013). Here however no detection of such a CC (even due to astrophysical processes) has been made so far, although this has already allowed for some constraints on DM properties to be derived (Shirasaki et al., 2016; Tröster et al., 2016). The main reason for the lack of detection is the too-small angular coverage of current weak-lensing surveys, and we might need to

[4]The DM decay signal would be proportional to DM overdensity, while the self-annihilation one to $\delta^2_{\rm DM}$.

[5]Note that the dominant component of the γ-ray background comes from our Galaxy and this also must be removed in analyses of cosmological context.

wait for future wide-angle weak-lensing observations such as from LSST or Euclid for the CC of cosmic shear with the γ-ray background to be detected.

4.5 Galaxy peculiar velocities

Most of the LSS statistics discussed so far rely on galaxy positions and their spatial density. Peculiar velocities are a complementary probe to galaxy densities are (see *e.g.* an excellent albeit a bit outdated review by Strauss & Willick, 1995). These can be inferred from measured galaxy spectroscopic redshifts together with redshift independent distance indicators. Commonly used methods rely on estimating galaxy absolute properties using calibrated galaxy scaling relations, such as Tully-Fisher (Tully & Fisher, 1977) for spiral galaxies or the Fundamental Plane for ellipticals (Djorgovski & Davis, 1987). For galaxies that host an observed SNIa event, distances can be also ascertained from standard supernova light curve models. Having an estimate of a galaxy distance, one can subtract the homogeneous Hubble recession velocity from the observed redshift and estimate the radial component of a galaxy peculiar velocity.

Recent years have witnessed the advent of high-quality and rich galaxy peculiar velocity data, *e.g.* the SFI++ (Springob et al., 2007), 6dF (Springob et al., 2014), and Cosmicflows (Courtois et al., 2011; Tully et al., 2013, 2016). This re-kindled activity in the peculiar velocity field, with the new data offering an unprecedented opportunity for cosmological measurements and theory testing. In cosmological perturbation theory (*e.g.* Peebles, 1980; Nusser et al., 1991), peculiar velocities are proportional to the gravitational force field. Therefore, peculiar velocity catalogues are direct probes of dark matter density and can in principle provide valuable information on fundamental theories for structure formation. However, the challenge lies in optimally handling the peculiar velocity data, which are relatively sparse and noisy, and are affected by significant errors associated with redshift-independent distance indicators. In addition, the methods used for inferring galaxy velocities introduce highly inhomogeneous and non-linear systematic effects (such as the Malmquist bias) that are difficult to model. A further disadvantage is that peculiar velocities can only be measured with sufficient accuracy for relatively small (as compared to redshift surveys) samples of local galaxies ($z < 0.05$)

The above-mentioned disadvantages of galaxy peculiar velocity data sets are, however, fully compensated by the fact that the velocity field offers, in principle, unbiased and model independent probes of both the nature of gravity and the distribution of dark matter in the Universe. Therefore the potential gain is tremendous as peculiar velocities can provide novel MG/GR tests that also constitute independent consistency checks for the GR framework to secure the cosmological parameters estimated from the traditional cosmological probes mentioned above.

The new data that will come in the near future from surveys like TAIPAN (Howlett et al., 2017), CosmicFlows-4, radio surveys (*e.g.* WALLABY Johnston et al., 2008)), and Gaia (Nusser et al., 2012, providing cosmological tangential velocities) will offer an unprecedented opportunity for new cosmological measurements and tests of theory. Peculiar velocities were shown to be a promising probe of the growth rate of structures (Koda et al., 2014; Johnson et al., 2014, 2016) and proven competitive with other probes, such as RSDs, (Hudson & Turnbull, 2012) especially at very low redshifts where possible deviations from GR due to MG should be the strongest (*e.g.* Hellwing et al., 2013).

4.6 Gravitational waves?

In 2016, a new window on the Universe opened thanks to the first direct detections of gravitational waves (GW) made by the LIGO observatory (Abbott et al., 2016b,a). Of interest for LSS studies is that both these early detections come from cosmological distances of $z \sim 0.1$. The detection rate of such events with LIGO and its planned extensions (aLIGO, VIRGO) will be too low to be useful for observational cosmology. However, future GW detectors, such as LISA or Einstein Telescope, could bring perhaps thousands of GW event detections and their precise localisations, which could provide new LSS maps using signals totally independent of the electromagnetic radiation. Such maps could be then used together with the 'classic' LSS data for various cosmological constraints (Cai & Yang, 2016; Oguri, 2016; Tamanini, 2016).

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References

Abbott, B. P., et al., *GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence*, *Physical Review Letters* **116**, 24, 241103 (2016a), 1606.04855

Abbott, B. P., et al., *Observation of Gravitational Waves from a Binary Black Hole Merger*, *Physical Review Letters* **116**, 6, 061102 (2016b), 1602.03837

Abdalla, F. B., et al., *Cosmology from HI galaxy surveys with the SKA*, *Advancing Astrophysics with the Square Kilometre Array (AASKA14)* 17 (2015), 1501.04035

Alonso, D., et al., *Homogeneity and isotropy in the Two Micron All Sky Survey Photometric Redshift catalogue*, MNRAS **449**, 670 (2015), 1412.5151

Anderson, M. E., Churazov, E., Bregman, J. N., *Non-Detection of X-Ray Emission From Sterile Neutrinos in Stacked Galaxy Spectra*, *ArXiv e-prints* (2014), 1408.4115

Aubourg, É., et al., *Cosmological implications of baryon acoustic oscillation measurements*, Phys. Rev. D **92**, 12, 123516 (2015), 1411.1074

Bartelmann, M., Schneider, P., *Weak gravitational lensing*, Phys. Rep. **340**, 291 (2001), astro-ph/9912508

Bassett, B., Hlozek, R., Baryon acoustic oscillations, 246 (2010)

Baugh, C. M., *A primer on hierarchical galaxy formation: the semi-analytical approach*, *Reports on Progress in Physics* **69**, 3101 (2006), astro-ph/0610031

Bennett, C. L., et al., *The Microwave Anisotropy Probe Mission*, ApJ **583**, 1 (2003), astro-ph/0301158

Bergström, L., Ullio, P., Buckley, J. H., *Observability of γ rays from dark matter neutralino annihilations in the Milky Way halo*, *Astroparticle Physics* **9**, 137 (1998), astro-ph/9712318

Beutler, F., et al., *The 6dF Galaxy Survey: baryon acoustic oscillations and the local Hubble constant*, MNRAS **416**, 3017 (2011), 1106.3366

Beutler, F., et al., *The 6dF Galaxy Survey: $z \simeq 0$ measurements of the growth rate and σ_8*, MNRAS **423**, 3430 (2012), 1204.4725

Bilicki, M., et al., *Two Micron All Sky Survey Photometric Redshift Catalog: A Comprehensive Three-dimensional Census of the Whole Sky*, ApJS **210**, 9 (2014), 1311.5246

Blake, C., Bridle, S., *Cosmology with photometric redshift surveys*, MNRAS **363**, 1329 (2005), astro-ph/0411713

Bleem, L. E., et al., *A Measurement of the Correlation of Galaxy Surveys with CMB Lensing Convergence Maps from the South Pole Telescope*, ApJ **753**, L9 (2012), 1203.4808

Boggess, N. W., et al., *The COBE mission - Its design and performance two years after launch*, ApJ **397**, 420 (1992)

Bond, J. R., Kofman, L., Pogosyan, D., *How filaments of galaxies are woven into the cosmic web*, Nature **380**, 603 (1996), astro-ph/9512141

Bose, S., et al., *The Copernicus Complexio: statistical properties of warm dark matter haloes*, MNRAS **455**, 318 (2016), 1507.01998

Boughn, S., Crittenden, R., *A correlation between the cosmic microwave background and large-scale structure in the Universe*, Nature **427**, 45 (2004), astro-ph/0305001

Boyarsky, A., Ruchayskiy, O., Iakubovskyi, D., Franse, J., *Unidentified Line in X-Ray Spectra of the Andromeda Galaxy and Perseus Galaxy Cluster*, Phys. Rev. Lett. **113**, 25, 251301 (2014), 1402.4119

Boyarsky, A., Ruchayskiy, O., Shaposhnikov, M., *The Role of Sterile Neutrinos in Cosmology and Astrophysics*, *Annu. Rev. Nucl. Part. Sci.* **59**, 191 (2009), 0901.0011

Buchert, T., et al., *Is there proof that backreaction of inhomogeneities is irrelevant in cosmology?*, *Classical and Quantum Gravity* **32**, 21, 215021 (2015), 1505.07800

Bulbul, E., et al., *Detection of an Unidentified Emission Line in the Stacked X-Ray Spectrum of Galaxy Clusters*, ApJ **789**, 13 (2014), 1402.2301

Busca, N. G., et al., *Baryon acoustic oscillations in the Lyα forest of BOSS quasars*, A&A **552**, A96 (2013), 1211.2616

Cai, R.-G., Yang, T., *Estimating cosmological parameters by the simulated data of gravitational waves from Einstein Telescope*, *ArXiv e-prints* (2016), 1608.08008

Camera, S., Fornasa, M., Fornengo, N., Regis, M., *A Novel Approach in the Weakly Interacting Massive Particle Quest: Cross-correlation of Gamma-Ray Anisotropies and Cosmic Shear*, ApJ **771**, L5 (2013), 1212.5018

Carroll, S. M., *The Cosmological Constant*, *Living Reviews in Relativity* **4**, 1 (2001), astro-ph/0004075

Carroll, S. M., Duvvuri, V., Trodden, M., Turner, M. S., *Is cosmic speed-up due to new gravitational physics?*, Phys. Rev. D **70**, 4, 043528 (2004), astro-ph/0306438

Clifton, T., Ferreira, P. G., Padilla, A., Skordis, C., *Modified gravity and cosmology*, Phys. Rep. **513**, 1 (2012), 1106.2476

Cole, S., et al., *The 2dF Galaxy Redshift Survey: power-spectrum analysis of the final data set and cosmological implications*, MNRAS **362**, 505 (2005), astro-ph/0501174

Courtois, H. M., et al., *Cosmic Flows: Green Bank Telescope and Parkes H I observations*, MNRAS **414**, 2005 (2011), 1101.3802

Crill, B. P., et al., *BOOMERANG: A Balloon-borne Millimeter-Wave Telescope and Total Power Receiver for Mapping Anisotropy in the Cosmic Microwave Background*, ApJS **148**, 527 (2003), astro-ph/0206254

Crittenden, R. G., Turok, N., *Looking for a Cosmological Constant with the Rees-Sciama Effect*, *Physical Review Letters* **76**, 575 (1996), astro-ph/9510072

Cuoco, A., et al., *Dark Matter Searches in the Gamma-ray Extragalactic Background via Cross-correlations with Galaxy Catalogs*, ApJS **221**, 29 (2015), 1506.01030 de Jong, J. T. A., et al., *The Kilo-Degree Survey*, *The Messenger* **154**, 44 (2013)

de la Torre, S., et al., *The VIMOS Public Extragalactic Redshift Survey (VIPERS) . Galaxy clustering and redshift-space distortions at z ∼= 0.8 in the first data release*, A&A **557**, A54 (2013), 1303.2622

Djorgovski, S., Davis, M., *Fundamental properties of elliptical galaxies*, ApJ **313**, 59 (1987)

Eisenstein, D. J., et al., *Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies*, ApJ **633**, 560 (2005), astro-ph/0501171

Frenk, C. S., White, S. D. M., *Dark matter and cosmic structure*, *Annalen der Physik* **524**, 507 (2012), 1210.0544

Górski, K. M., et al., *HEALPix: A Framework for High-Resolution Discretization and Fast Analysis of Data Distributed on the Sphere*, ApJ **622**, 759 (2005), astro-ph/0409513

Green, S. R., Wald, R. M., *Comments on Backreaction*, *ArXiv e-prints* (2015), 1506.06452

Guzzo, L., *Is the universe homogeneous? (On large scales)*, New A **2**, 517 (1997), astro-ph/9711206

Guzzo, L., The Vipers Team, *VIPERS: An Unprecedented View of Galaxies and Large-scale Structure Halfway Back in the Life of the Universe*, *The Messenger* **151**, 41 (2013), 1303.3930

Hamana, T., Takada, M., Yoshida, N., *Searching for massive clusters in weak lensing surveys*, MNRAS **350**, 893 (2004), astro-ph/0310607

Hand, N., et al., *First measurement of the cross-correlation of CMB lensing and galaxy lensing*, Phys. Rev. D **91**, 6, 062001 (2015), 1311.6200

Harvey, D., et al., *The nongravitational interactions of dark matter in colliding galaxy clusters*, *Science* **347**, 1462 (2015), 1503.07675

Hellwing, W. A., Li, B., Frenk, C. S., Cole, S., *Hierarchical clustering in chameleon f(R) gravity*, MNRAS **435**, 2806 (2013), 1305.7486

Hellwing, W. A., Nusser, A., Feix, M., Bilicki, M., *Not a Copernican observer: biased peculiar velocity statistics in the local Universe*, *ArXiv e-prints* (2016), 1609.07120

Hellwing, W. A., et al., *Clear and Measurable Signature of Modified Gravity in the Galaxy Velocity Field*, Phys. Rev. Lett. **112**, 22, 221102 (2014), 1401.0706

Heymans, C., et al., *CFHTLenS: the Canada-France-Hawaii Telescope Lensing Survey*, MNRAS **427**, 146 (2012), 1210.0032

Hoekstra, H., Jain, B., *Weak Gravitational Lensing and Its Cosmological Applications*, *Annual Review of Nuclear and Particle Science* **58**, 99 (2008), 0805.0139

Hogg, D. W., et al., *Cosmic Homogeneity Demonstrated with Luminous Red Galaxies*, ApJ **624**, 54 (2005), astro-ph/0411197

Hooper, D., Goodenough, L., *Dark matter annihilation in the Galactic Center as seen by the Fermi Gamma Ray Space Telescope*, *Phys. Lett. B* **697**, 412 (2011), 1010.2752

Howlett, C., Staveley-Smith, L., Blake, C., *Cosmological forecasts for combined and next-generation peculiar velocity surveys*, MNRAS **464**, 2517 (2017), 1609.08247

Huchra, J. P., et al., *The 2MASS Redshift Survey – Description and Data Release*, ApJS **199**, 26 (2012), 1108.0669

Hudson, M. J., Turnbull, S. J., *The Growth Rate of Cosmic Structure from Peculiar Velocities at Low and High Redshifts*, ApJ **751**, L30 (2012), 1203.4814

Jain, B., Khoury, J., *Cosmological tests of gravity*, *Annals of Physics* **325**, 1479 (2010), 1004.3294

Jain, B., Zhang, P., *Observational tests of modified gravity*, Phys. Rev. D **78**, 6, 063503 (2008), 0709.2375

Johnson, A., et al., *The 6dF Galaxy Survey: cosmological constraints from the velocity power spectrum*, MNRAS **444**, 3926 (2014), 1404.3799

Johnson, A., et al., *Searching for modified gravity: scale and redshift dependent constraints from galaxy peculiar velocities*, MNRAS **458**, 2725 (2016), 1504.06885

Johnston, S., et al., *Science with ASKAP. The Australian square-kilometre-array pathfinder*, *Experimental Astronomy* **22**, 151 (2008), 0810.5187

Joyce, A., Jain, B., Khoury, J., Trodden, M., *Beyond the cosmological standard model*, Phys. Rep. **568**, 1 (2015), 1407.0059

Joyce, A., Lombriser, L., Schmidt, F., *Dark Energy Versus Modified Gravity*, *Annu. Rev. Nucl. Part. Sci.* **66**, 95 (2016), 1601.06133

Kaiser, N., *Clustering in real space and in redshift space*, MNRAS **227**, 1 (1987)

Kilbinger, M., *Cosmology with cosmic shear observations: a review*, *Reports on Progress in Physics* **78**, 8, 086901 (2015), 1411.0115

Kim, S. Y., Peter, A. H. G., Wittman, D., *In the Wake of Dark Giants: New Signatures of Dark Matter Self Interactions in Equal Mass Mergers of Galaxy Clusters*, *ArXiv e-prints* (2016), 1608.08630

Kirk, D., et al., *Cross-correlation of gravitational lensing from DES Science Verification data with SPT and Planck lensing*, MNRAS **459**, 21 (2016), 1512.04535

Koda, J., et al., *Are peculiar velocity surveys competitive as a cosmological probe?*, MNRAS **445**, 4267 (2014), 1312.1022

Koyama, K., *Cosmological tests of modified gravity*, *Reports on Progress in Physics* **79**, 4, 046902 (2016), 1504.04623

Kratochvil, J. M., et al., *Probing cosmology with weak lensing Minkowski functionals*, Phys. Rev. D **85**, 10, 103513 (2012), 1109.6334

Laureijs, R., et al., *Euclid Definition Study Report*, *ArXiv e-prints* (2011), 1110.3193

Levi, M., et al., *The DESI Experiment, a whitepaper for Snowmass 2013*, *ArXiv e-prints* (2013), 1308.0847

Lewis, A., Challinor, A., *Weak gravitational lensing of the CMB*, Phys. Rep. **429**, 1 (2006), astro-ph/0601594

Li, R., et al., *Constraints on the identity of the dark matter from strong gravitational lenses*, MNRAS **460**, 363 (2016), 1512.06507

Lovell, M. R., et al., *The haloes of bright satellite galaxies in a warm dark matter universe*, MNRAS **420**, 2318 (2012), 1104.2929

LSST Science Collaboration, et al., *LSST Science Book, Version 2.0*, *ArXiv e-prints* (2009), 0912.0201

Maartens, R., *Is the Universe homogeneous?*, *Philosophical Transactions of the Royal Society of London Series A* **369**, 5115 (2011), 1104.1300

Malyshev, D., Neronov, A., Eckert, D., *Constraints on 3.55 keV line emission from stacked observations of dwarf spheroidal galaxies*, Phys. Rev. D **90**, 10, 103506 (2014), 1408.3531

Mo, H., van den Bosch, F. C., White, S., Galaxy Formation and Evolution (2010)

Nusser, A., Branchini, E., Davis, M., *Gaia: A Window to Large-scale Motions*, ApJ **755**, 58 (2012), 1202.4138

Nusser, A., Dekel, A., Bertschinger, E., Blumenthal, G. R., *Cosmological velocity-density relation in the quasi-linear regime*, ApJ **379**, 6 (1991)

Oguri, M., *Measuring the distance-redshift relation with the cross-correlation of gravitational wave standard sirens and galaxies*, Phys. Rev. D **93**, 8, 083511 (2016), 1603.02356

Pan, J., Coles, P., *Large-scale cosmic homogeneity from a multifractal analysis of the PSCz catalogue*, MNRAS **318**, L51 (2000), astro-ph/0008240

Pandey, B., Sarkar, S., *Testing homogeneity in the Sloan Digital Sky Survey Data Release Twelve with Shannon entropy*, MNRAS **454**, 2647 (2015), 1507.03124

Pandey, B., Sarkar, S., *Probing large scale homogeneity and periodicity in the LRG distribution using Shannon entropy*, MNRAS **460**, 1519 (2016), 1512.06350

Peebles, P. J., Ratra, B., *The cosmological constant and dark energy*, *Reviews of Modern Physics* **75**, 559 (2003), astro-ph/0207347

Peebles, P. J. E., The large-scale structure of the universe (1980)

Perlmutter, S., et al., *Measurements of Ω and Λ from 42 High-Redshift Supernovae*, ApJ **517**, 565 (1999), astro-ph/9812133

Planck Collaboration, et al., *Planck 2013 results. XVII. Gravitational lensing by large-scale structure*, A&A **571**, A17 (2014), 1303.5077

Planck Collaboration, et al., *Planck 2015 results. XI. CMB power spectra, likelihoods, and robustness of parameters*, A&A **594**, A11 (2016a), 1507.02704

Planck Collaboration, et al., *Planck 2015 results. XIII. Cosmological parameters*, A&A **594**, A13 (2016b), 1502.01589

Planck Collaboration, et al., *Planck 2015 results. XVI. Isotropy and statistics of the CMB*, A&A **594**, A16 (2016c), 1506.07135

Planck Collaboration, et al., *Planck 2015 results. XVII. Constraints on primordial non-Gaussianity*, A&A **594**, A17 (2016d), 1502.01592

Pullen, A. R., Alam, S., He, S., Ho, S., *Constraining gravity at the largest scales through CMB lensing and galaxy velocities*, MNRAS **460**, 4098 (2016), 1511.04457

Rees, M. J., Sciama, D. W., *Large-scale Density Inhomogeneities in the Universe*, Nature **217**, 511 (1968)

Refregier, A., *Weak Gravitational Lensing by Large-Scale Structure*, ARA&A **41**, 645 (2003), astro-ph/0307212

Riemer-Sorensen, S., *Questioning a 3.5 keV dark matter emission line*, *ArXiv e-prints* (2014), 1405.7943

Riess, A. G., et al., *Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant*, AJ **116**, 1009 (1998), astro-ph/9805201

Sachs, R. K., Wolfe, A. M., *Perturbations of a Cosmological Model and Angular Variations of the Microwave Background*, ApJ **147**, 73 (1967)

Samushia, L., Percival, W. J., Raccanelli, A., *Interpreting large-scale redshift-space distortion measurements*, MNRAS **420**, 2102 (2012), 1102.1014

Schaye, J., et al., *The EAGLE project: simulating the evolution and assembly of galaxies and their environments*, MNRAS **446**, 521 (2015), 1407.7040

Scrimgeour, M. I., et al., *The WiggleZ Dark Energy Survey: the transition to large-scale cosmic homogeneity*, MNRAS **425**, 116 (2012), 1205.6812

Sherwin, B. D., et al., *The Atacama Cosmology Telescope: Cross-correlation of cosmic microwave background lensing and quasars*, Phys. Rev. D **86**, 8, 083006 (2012), 1207.4543

Shirasaki, M., et al., *Cosmological constraints on dark matter annihilation and decay: Cross-correlation analysis of the extragalactic γ -ray background and cosmic shear*, Phys. Rev. D **94**, 6, 063522 (2016), 1607.02187

Somerville, R. S., Davé, R., *Physical Models of Galaxy Formation in a Cosmological Framework*, ARA&A **53**, 51 (2015), 1412.2712

Springob, C. M., et al., *SFI++. II. A New I-Band Tully-Fisher Catalog, Derivation of Peculiar Velocities, and Data Set Properties*, ApJS **172**, 599 (2007)

Springob, C. M., et al., *The 6dF Galaxy Survey: peculiar velocity field and cosmography*, MNRAS **445**, 2677 (2014), 1409.6161

Steigman, G., *Primordial Nucleosynthesis in the Precision Cosmology Era*, *Annu. Rev. Nucl. Part. Sci.* **57**, 463 (2007), 0712.1100

Strauss, M. A., Willick, J. A., *The density and peculiar velocity fields of nearby galaxies*, Phys. Rep. **261**, 271 (1995), astro-ph/9502079

Takada, M., *Subaru Hyper Suprime-Cam Project*, in N. Kawai, S. Nagataki (eds.) American Institute of Physics Conference Series, *American Institute of Physics Conference Series*, volume 1279, 120–127 (2010)

Tamanini, N., *Late time cosmology with LISA: probing the cosmic expansion with massive black hole binary mergers as standard sirens*, *ArXiv e-prints* (2016), 1612.02634

Tauber, J. A., et al., *Planck pre-launch status: The Planck mission*, A&A **520**, A1 (2010)

The COrE Collaboration, et al., *COrE (Cosmic Origins Explorer) A White Paper*, *ArXiv e-prints* (2011), 1102.2181

The Dark Energy Survey Collaboration, *The Dark Energy Survey*, *ArXiv Astrophysics e-prints* (2005), astro-ph/0510346

Tröster, T., et al., *Cross-correlation of weak lensing and gamma rays: implications for the nature of dark matter*, *ArXiv e-prints* (2016), 1611.03554

Tully, R. B., Courtois, H. M., Sorce, J. G., *Cosmicflows-3*, *ArXiv e-prints* (2016), 1605.01765

Tully, R. B., Fisher, J. R., *A new method of determining distances to galaxies*, A&A **54**, 661 (1977)

Tully, R. B., et al., *Cosmicflows-2: The Data*, AJ **146**, 86 (2013), 1307.7213

Vogelsberger, M., et al., *Introducing the Illustris Project: simulating the coevolution of dark and visible matter in the Universe*, MNRAS **444**, 1518 (2014), 1405.2921

Will, C. M., *The Confrontation between General Relativity and Experiment*, *Living Reviews in Relativity* **17**, 1, 4 (2014), URL http://dx.doi.org/10.12942/lrr-2014-4

Wu, K. K. S., Lahav, O., Rees, M. J., *The large-scale smoothness of the Universe*, Nature **397**, 225 (1999), astro-ph/9804062

Xia, J.-Q., Cuoco, A., Branchini, E., Viel, M., *Tomography of the Fermi-LAT γ-Ray Diffuse Extragalactic Signal via Cross Correlations with Galaxy Catalogs*, ApJS **217**, 15 (2015), 1503.05918

Yadav, J. K., Bagla, J. S., Khandai, N., *Fractal dimension as a measure of the scale of homogeneity*, MNRAS **405**, 2009 (2010), 1001.0617