Cosmology with strong lensing systems

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Strong gravitational lensing has now developed into a mature tool for investigating galactic structure and dynamics as well as cosmological models. In this lecture the phenomenon of strong gravitational lensing, its history and applications are reviewed with an emphasis on the recent ideas developed by the author. Expected massive discoveries of strong lensing galactic scale systems in forthcoming projects like Euclid or LSST herald the bright future of gravitational lensing in cosmology.

1 Introduction

Strong lensing is the phenomenon stemming from light bending by massive bodies predicted by General Relativity. According to General Relativity, matter, energy and their flows, physically described by the energy-momentum tensor $T_{\mu\nu}$, curve spacetime according to the Einstein equations: $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$, where $G_{\mu\nu}$ (called the Einstein tensor) describes the curvature of spacetime. Consequently, free particles and light no longer move along straight lines but rather along geodesics in the curved spacetime. Soon after formulating General Relativity, when the solution (due to Schwarzschild) for the metric outside a static, spherically symmetric mass distribution (like a star) was known, Einstein was able to show that the light ray coming from a distant source and passing by the mass M at the closest encounter distance (impact parameter) b is deflected by an angle equal to: $\alpha = \frac{4GM}{c^2 b}$.

Earlier, the deflection of light by massive bodies was discussed in the framework of Newtonian gravity with the result of $\alpha = \frac{2GM}{c^2 b}$. This result is usually attributed to Soldner who published it in 1801, although historical records point to Henry Cavendish who obtained the same result around 1784 inspired by a letter from John Michell. Einstein unaware of this recovered the Newtonian prediction in 1911 on the ground of equivalence principle, assuming however that space is Euclidean. In the standard setting of a light ray grazing the Sun, i.e. with $b = R_{\odot}$, the deflection angle according to the Newtonian scheme is $0''.875$. Hence, from the observational point of view it was fortunate that relativistic prediction of $1''.75$ was bigger by a factor of two. This was easier to observe and was indeed confirmed in 1919 by Sir Arthur Eddington during the solar eclipse that took place in front of the Hyades cluster. Soon thereafter, Eddington, Chwolson, and Einstein himself realized that light bending phenomenon is able to produce multiple images of stars lying farther away than stars acting as lenses. Of course almost perfect alignment between the lens and the source is necessary for this. However, more detailed calculations using a solar mass value as typical for stars, and distances of order $5 - 10$ kpc as typical for our Galaxy, imply that the images should be separated by about $0''.001$, which of course made Einstein and contemporary astronomers very sceptical regarding the observability of such an effect.

On the other hand, Zwicky (1937) has argued that galaxies having masses of order of $10^{11} - 10^{12}\ M_{\odot}$, even though separated by distances of 10 Mpc−1 Gpc, could act as gravitational lenses producing multiple images of more distant objects separated by about $1''$. His early intuition turned out to be true but we had to wait until the seventies of the $20^{\rm th}$ century when Walsh et al. (1979) serendipitously discovered a lensed quasar — the first gravitational lens. The rich and interesting history of gravitational lensing from these early phases up to modern times can be found in Schneider et al. (1992, chap. 1.1).

2 Strong lensing theory in brief

There exist a number of very good textbooks devoted to gravitational lensing theory, which became a mature discipline with its own terminology and formalism. For an excellent introduction to this theory, the reader is encouraged to look into the now classic books by Schneider et al. (1992) and (Meylan et al., 2006). Quick, yet comprehensive starters are offered by the lecture notes from the 1995 Jerusalem Winter School (Narayan & Bartelmann, 1996) and from Massimo Meneghetti[1]. This section will briefly introduce the main ideas and terminology used further in this paper.

Imagine the source, observer and a massive object (the lens) placed exactly along a line (the optical bench). From the point of view of classical optics the source would be obscured by the intervening object: the only light ray (or a small collimated bunch of rays) pointing toward the observer would not reach him. The general relativistic phenomenon of light deflection near massive bodies changes this picture: out of all light rays emitted radially some of them (passing close enough by the deflector – how close depends on the mutual locations of source, deflector and observer) are now focused at the observer. The intervening massive body acts as a lens and a source behind it reveals its existence as a luminous ring — the so called Einstein ring. Even the smallest misalignment of the source, the lens and observer results typically in multiple images whose angular positions and magnification ratios allow reconstruction of the lensing mass distribution.

As in classical optics, there are two equivalent approaches to understand the phenomenon: the light-rays formalism and the wavefronts formalism. From the point of view of Fermat's principle, the light travel time can be calculated as

$$t(\mathbf{x}) = \frac{1+z_l}{c}\frac{D_l D_s}{D_{ls}}\left[\frac{1}{2}\left(\mathbf{x}-\beta\right)^2 - \psi(\mathbf{x})\right] \tag{1}$$

where: $\mathbf{x}$ and β are positions (as projected on the celestial sphere) of the image and the source, D_l, D_s are angular diameter distances to the lens and to the source located at redshifts z_l and z_s respectively (D_{ls} is the angular diameter distance between lens and source), $\psi(\mathbf{x})$ is the projected gravitational potential (i.e. the actual potential Φ integrated along the line of sight) satisfying two dimensional Poisson equation:

$$\triangle\psi = 2\kappa \tag{2}$$

where κ is the (projected) surface mass density in units of critical density $\Sigma_{cr} = c^2 D_s/(4\pi G D_l D_{ls})$. Then, Fermat's principle states that images form at stationary points of the time delay surface $\nabla t(\mathbf{x})$, which leads to the lens equation:

$$\beta = \mathbf{x} - \nabla\psi = \mathbf{x} - \alpha \tag{3}$$

[1] http://www.ita.uni-heidelberg.de/ massimo/sub/Lectures/gl_all.pdf

The last equality is usually invoked in the light-rays formalism, where $\alpha = \frac{D_{ls}}{D_s}\hat{\alpha}$ is scaled deflection angle. In axially symmetric lenses, for example: $\hat{\alpha}(\mathbf{x}) = \frac{4GM(x)}{c^2 x^2}\mathbf{x}$ where $M(x)$ is the mass enclosed by the circle of radius $x = |\mathbf{x}|$. The most useful notion in gravitational lensing theory is the Einstein radius ϑ_E, i.e. the radius of the circle inside which the average projected mass density is equal to the critical density (cf. above). Thus the Einstein radius defines the deflection scale of a given lens.

The lensing is called strong if the source position happens to lie within the circle of radius ϑ_E. In this case multiple images appear. In the opposite case (i.e. the light-rays from the source pass by the lens outside its Einstein radius) there are no multiple images. However, even in this case light-ray bundle experiences systematic distortion which changes the shape of the lensed image of the source. This phenomenon, called weak lensing, has become an important tool in modern cosmology (Meylan et al., 2006). It is, however, beyond the scope of this lecture.

Since lensing galaxies are mostly ellipticals, the number of images is often equal to four – the issue of image multiplicity is discussed e.g. by Schneider et al. (1992). However, a surprisingly realistic model of the lens potential is that of a singular isothermal sphere (SIS) in which the 3-dimensional mass density has the following profile:

$$\rho = \frac{\sigma_{SIS}^2}{2\pi G r^2} \tag{4}$$

Indeed, lensing by ellipticals can be modeled by its variant called 'singular isothermal ellipsoid' (SIE). Therefore for the illustrative purposes it would be sufficient to restrict our attention to the SIS model. Other realistic and more sophisticated models are discussed in classical textbooks (Schneider et al., 1992).

The Einstein ring radius for the SIS model is:

$$\vartheta_E = 4\pi \frac{D_{ls}}{D_s}\frac{\sigma^2}{c^2} \tag{5}$$

where σ denotes one-dimensional velocity dispersion of stars in lensing galaxy. If the lensing is strong i.e. $\beta < \vartheta_E$ then two co-linear images A and B form on the opposite side of the lens, at radial distances $\vartheta_A = \vartheta_E + \beta$ and $\vartheta_B = \vartheta_E - \beta$.

Besides multiple images, another important ingredient of gravitational lensing is the time delay between lensed images of the source. Light rays from these images travel along paths differing in length and probe the gravitational potential of the lens at different depths, thus experiencing different gravitational time delays. These two effects, the geometrical and the Shapiro effect, combine to produce the time delay between images. This can be best understood in terms of Fermat's principle, in other words, the intervening mass between the source and the observer introduces an effective index of refraction, thereby increasing the light travel time. In the aforementioned SIS model, time delay between the images is:

$$\Delta t_{SIS} = \frac{1+z_l}{2c}\frac{D_l D_s}{D_{ls}}(\vartheta_A^2 - \vartheta_B^2) \tag{6}$$

which according to the above mentioned relations for the SIS model can also be written as

$$\Delta t_{SIS} = \frac{2(1+z_l)}{c}\frac{D_l D_s}{D_{ls}}\vartheta_E \beta = \frac{8\pi}{H_0}\tilde{r}_l \beta \frac{\sigma^2}{c^2} \tag{7}$$

In the last equation $\tilde{r}_l$ denotes the reduced (non-dimensional) comoving distance to the lens. Equation (6) is commonly used by the gravitational lensing community because it reduces time delay problem to relative astrometry of images, whereas β is much harder to asses (it must be small in order for strong lensing to occur) and the Einstein ring radius is not a directly observable quantity (although image separation fairly represents the Einstein radius). Equation (7) is sometimes more useful from the theoretical point of view.

The last observable derivable from strongly lensed systems is the flux ratio of images. It is the most sensitive with respect to details of mass distribution along the light-ray path both in terms of detailed knowledge of the smooth component of the mass distribution as well as the graininess of the lens, i.e. microlensing by stars or other clumped massive structures along the path.

3 Cosmology with strong lensing systems.

From the first discovery of strong lensing system until the end of the $20^{\rm th}$ century, searches were focused on potential sources (quasars) seeking for close pairs or multiples and checking whether there exists an intervening galaxy acting as a lens. This strategy turned out not to be particularly efficient — up to 1992 there were only a dozen strong lensing sytems known. Moreover if one wanted to study such a strong lensing system in more detailed way, one was forced to gain more knowledge about the lens from separate observational sessions focused on the lensing galaxy.

A new epoch started with dedicated surveys like Sloan Lens ACS Survey (SLACS[2]). The SLACS survey (Bolton et al., 2006), unlike previous attempts, is focused on a possible lens population — massive ellipticals. The strong lensing cross section is proportional to the area of the Einstein ring $\sigma_{lens} = \pi\vartheta_E^2$, which means that the mass of the lens is a dominating factor. This is the main reason why in the vast majority of cases the lens is an E/SO type galaxy. This can be understood since ellipticals, being latecomers in hierarchical structure formation, are created in mergers of low-mass spiral galaxies. Hence they are more massive than spirals and the probability of their acting as lenses is higher. The Sloan Lens ACS Survey (SLACS) and the BOSS emission-line lens survey (BELLS) are spectroscopic lens surveys in which candidates are selected respectively from the Sloan Digital Sky Survey (SDSS-III) data and Baryon Oscillation Spectroscopic Survey (BOSS). BOSS was initiated by upgrading the SDSS-I optical spectrographs (Eisenstein et al., 2011). The idea is to take the spectra of early type galaxies and to look for the presence of emission lines at redshift higher than that of the target galaxy. Candidates selected this way are followed-up with HST ACS snapshot imaging and after image processing (subtraction of the de Vaucouleurs profile of the target galaxy) those displaying multiple images and/or Einstein rings are classified as confirmed lenses.

3.1 Hubble constant from time delays

The first theoretical proposal of a serious cosmological application of strong gravitational lensing was presented by Refsdal (1964) in his stimulating paper on measurements of the Hubble constant from time delays between images. Namely, if the lensed source is intrinsically variable (quasars being the main population of sources displaying such variability) and we are able to extract the variability pattern from the light-curve (which in practice is non-trivial task), this variability would be observed at different times in the images. Then the time-delay, as e.g. (6) or better

[2] http://www.slacs.org/

yet (7), depends on the image locations and relative distances of the source, lens and observer. But the magnitude of this delay (the temporal scale of the effect) is set by H_0^{-1}. This creates the alternative possibility of measuring the Hubble constant H_0 which is, unlike other methods, independent of the cosmic distance ladder and its calibration. The number of lenses with reliably measured time delays has accumulated slowly over decades. Several years ago there were about 10 such lenses and the observational status of the Hubble constant determination, as reviewed in details in (Meylan et al., 2006), was that time delays preferred $H_0 = 52 \pm 6$ km s^{-1} Mpc^{-1} in contrast to the HST Key Project value of $H_0 = 72 \pm 8$ km s^{-1} Mpc^{-1} (Freedman et al., 2001). Later papers of Oguri et al. (2008) and Coles (2008) announced the results ($H_0 = 68 \pm 16$ km s^{-1} Mpc^{-1} and $H_0 = 71 \pm 8$ km s^{-1} Mpc^{-1} respectively) now in agreement with the HST value. However, the secondary lensing by galaxies located close to the optical axis of the main strong lensing system is the major source of confusion in the time delay method and it is very hard to estimate it properly. More recent analysis of B1608+656 by Suyu et al. (2010) explicitly took into account the weak lensing effects of external structures. They proposed to compare the B1608+656 field with a large number of fields with similar galaxy number overdensity drawn from the Millennium Simulation, modeling the line of sight effects with a single external convergence parameter. The most recent analysis of RXJ1131 strong lensing system has been performed in Suyu et al. (2013) where the authors also took into account the inferred external shear from the lens model reaching a significant improvement in precision.

3.2 Testing dark energy models: strong lensing systems as standard rulers

One of the most important issues in modern cosmology is the accelerated expansion of the universe, deduced from Type Ia supernovae but also confirmed by other independent probes, such as Cosmic Microwave Background (CMB) and the Large Scale Structure (LSS). In order to explain this phenomenon, a new component, called dark energy, which fuels the cosmic acceleration due to its negative pressure and may dominate the Universe at late times, has been introduced. Although cosmological constant Λ, the simplest candidate for dark energy, seems to fit in with current observations, yet it suffers from the well-known fine tuning and coincidence problems. Therefore a variety of dark energy models, including different dark energy equations of state (EoS) parametrizations such as XCDM model, and Chevalier-Polarski-Linder (CPL) model (Chevallier & Polarski, 2001; Linder, 2003) have been put forward, each of which has its own advantages and problems in explaining the acceleration of the universe. Yet, the nature of dark energy still remains unknown. It might also be possible that the observed accelerated expansion of the Universe is due to departures of the true theory of gravity from General Relativity, e.g. due to quantum nature of gravity or possible multidimensionality of the world. Hence such models like Dvali-Gabadadze-Porrati – inspired by brane theory or Ricci dark energy inspired by the holographic principle have been proposed. Having no clear preference from the side of theory and in order to learn more about dark energy, we have to turn to the sequential upgrading of observational fits of quantities which parametrize the unknown properties of dark energy (such as density parameters or coefficients in the cosmic equation of state) and seeking coherence among alternative tests. Strong lensing systems offer such a probe complementary to more standard ones like SNIa, CMB or BAO.

The idea of using strong lensing for measuring the cosmic equation of state was

was first proposed in the papers of (Yamamoto & Futamase, 2001; Biesiada, 2006) and further developed in (Biesiada et al., 2010; Cao et al., 2012, 2015) The main idea is that the formula for the Einstein radius in a SIS lens

$$\theta_E = 4\pi \frac{\sigma_{SIS}^2}{c^2} \frac{D_{ls}}{D_s} \tag{8}$$

depends on the cosmological model through the ratio of (angular-diameter) distances between lens and source and between observer and lens. Provided one has reliable knowledge about the lensing system: i.e. the Einstein radius θ_E (from image astrometry) and stellar velocity dispersion σ_{SIS} (from the central velocity dispersion σ_0 obtained from spectroscopy), one can use it to test the background cosmology. This method is independent of the Hubble constant value (which gets canceled in the distance ratio) and is not affected by dust absorption or source evolutionary effects. It depends, however, on the reliability of lens modeling (e.g. SIS or SIE assumption) and measurements of σ_0. Hopefully, starting with the Lens Structure and Dynamics (LSD) survey and the more recent SLACS survey spectroscopic data for central parts of lens galaxies became available allowing to assess their central velocity dispersions. There is growing evidence for the homologous structure of late type galaxies (Treu et al., 2006; Koopmans et al., 2006, 2009) supporting reliability of the SIS/SIE assumption. In particular it was shown there that, inside one effective radius, massive elliptical galaxies are kinematically indistinguishable from an isothermal ellipsoid.

In the method outlined above, the cosmological model parameters **p** enter not through a distance measure directly, but rather through a distance ratio

$$\mathcal{D}^{th}(z_l, z_s; \mathbf{p}) = \frac{D_s(\mathbf{p})}{D_{ls}(\mathbf{p})} = \frac{\int_0^{z_s} \frac{dz'}{h(z';\mathbf{p})}}{\int_{z_l}^{z_s} \frac{dz'}{h(z';\mathbf{p})}} \tag{9}$$

and the respective observable counterpart reads:

$$\mathcal{D}^{obs} = \frac{4\pi\sigma_0^2}{c^2\theta_E}$$

The positive side is that the Hubble constant H_0 gets canceled, hence it does not introduce any uncertainty to the results. On the other hand we have a disadvantage that the power of estimating Ω_m is poor.

In the recent paper Cao et al. (2015) generalized the SIS model to spherically symmetric power-law mass distribution $\rho \sim r^{-\gamma}$. Let us recall that the knowledge of θ_E (obtained from the location of observed images) provides us with the mass M_{lens} inside the Einstein radius: $M_{lens} = \pi R_E^2 \Sigma_{cr}$ where: $R_E = \theta_E D_l$ is the physical Einstein radius (in [kpc]) in the lens plane and Σ_{cr} is the critical projected mass density for lensing. Hence, we have:

$$M_{lens} = \frac{c^2}{4G} \frac{D_s D_l}{D_{ls}} \theta_E^2 \tag{10}$$

If one has spectroscopic data providing the velocity dispersion σ_{ap} inside the aperture (more precisely, luminosity averaged line-of-sight velocity dispersion), then after solving the spherical Jeans equation (assuming that stellar and mass distribution follow the same power-law and anisotropy vanishes) one can assess the dynamical mass inside the aperture projected onto the lens plane and scale it to the Einstein radius:

$$\begin{aligned} M_{dyn} &= \frac{\pi}{G}\sigma_{ap}^2 R_E \left(\frac{R_E}{R_{ap}}\right)^{2-\gamma} f(\gamma) \\ &= \frac{\pi}{G}\sigma_{ap}^2 D_l \theta_E \left(\frac{\theta_E}{\theta_{ap}}\right)^{2-\gamma} f(\gamma), \end{aligned} \tag{11}$$

where

$$\begin{aligned} f(\gamma) &= -\frac{1}{\sqrt{\pi}}\frac{(5-2\gamma)(1-\gamma)}{3-\gamma}\frac{\Gamma(\gamma-1)}{\Gamma(\gamma-3/2)} \\ &\times \left[\frac{\Gamma(\gamma/2-1/2)}{\Gamma(\gamma/2)}\right]^2. \end{aligned} \tag{12}$$

By combining Eq. (10) and Eq. (11), we obtain

$$\theta_E = 4\pi\frac{\sigma_{ap}^2}{c^2}\frac{D_{ls}}{D_s}\left(\frac{\theta_E}{\theta_{ap}}\right)^{2-\gamma} f(\gamma). \tag{13}$$

Now, our observable is

$$\mathcal{D}^{obs} = \frac{c^2\theta_E}{4\pi\sigma_{ap}^2}\left(\frac{\theta_{ap}}{\theta_E}\right)^{2-\gamma} f^{-1}(\gamma) \tag{14}$$

and its theoretical counterpart (the distance ratio) $\mathcal{D}^{th}(z_l, z_s; \mathbf{p})$ is given by Eq.(9). Velocity dispersions measured within an aperture should be transformed to velocity dispersions within circular aperture of radius $R_{\rm eff}/2$ (half the effective radius) following the prescription: $\sigma_0 = \sigma_{ap}(\theta_{\rm eff}/(2\theta_{ap}))^{-0.04}$. In the literature it has also been denoted as σ_{e2}.

In order to implement the methodology described above, we have made a comprehensive compilation of 118 strong lensing systems from four surveys: SLACS (57 lenses), BELLS (25 lenses), LSD (5 lenses), and SL2S (31 lenses). The LSD (Lens Structure and Dynamics) survey was a predecessor of SLACS in the sense that combined image and lens velocity dispersion data were used to constrain the structure of lensing galaxies. Because it was much earlier and lenses were selected optically (as multiple images of sources with identified lensing galaxies) and then followed up spectroscopically, only five systems from this survey comply with SLACS and BELLS. The last one — SL2S — is a project dedicated to finding galaxy scale lenses in the Canada-France-Hawaii Telescope Legacy Survey. The targets are massive red galaxies, and an automated *RingFinder* software is looking for tangentially elongated blue features around them. If found, they are followed up with HST and spectroscopy.

In our fits, the mass density power-law index γ was taken as a free parameter fitted together with cosmological parameters. It has been suggested by Ruff et al. (2011) that the mass density power-law index γ of massive elliptical galaxies evolves with redshift. Therefore in our fits we also assumed that the power-law mass density profile can evolve: $\gamma(z_l) = \gamma_0 + \gamma_1 z_l$. Concerning a non-evolving slope $\gamma = const$ the best fit was $\gamma = 2.07 \pm 0.07$ i.e. close to isothermal, we found that the dark energy equation of the state parameter was $w = -1.15^{+0.56}_{-1.20}$ which agreed very well with the respective value derived from *Planck* observations combined with BAO data, i.e. $w = -1.13^{+0.13}_{-0.10}$. Allowing the mass density profile to evolve with redshift, the constraints on the mass density power-law index parameters $\gamma_0 = 2.13^{+0.07}_{-0.12}$,

$\gamma_1 = -0.09 \pm 0.17$ were also consistent with the previous analysis of (Ruff et al., 2011) who reported: $\gamma(z_l) = 2.12^{+0.03}_{-0.04} - 0.25^{+0.10}_{-0.12} \times z_l + 0.17^{+0.02}_{-0.02}(scatter)$. The cosmic equation of state was then assessed as $w = -1.35^{+0.67}_{-1.50}$ again in agreement with *Planck*. In another paper (Li et al., 2016) the same method was used to a wider class of cosmological models, comprising also the DGP braneworld scenario and Ricci dark energy besides the ΛCDM, quintessence and CPL. It turned out that the best fitted mass density slopes γ were in agreement with each other, irrespective of the cosmological model considered. This demonstrates that the method of using galactic strong lensing systems as standard rulers can be further refined and eventually may provide a complementary probe to test the properties of dark energy.

There are still several sources of systematics which should be considered in the future. The first one is related to the interpretation of observed velocity dispersions. Namely, the spherical Jeans equation has been adopted to connect the observed velocity dispersions to the masses, and this was done assuming that anisotropy β parameter was zero. One can show that the anisotropy parameter is degenerate with the slope γ. Therefore, in the approach pursued so far, the power-law index should understood as an effective descriptor capturing both the density profile and anisotropy of the velocity dispersions. Another issue is the three-dimensional shape of lensing galaxies, the prolateness/oblateness of lensing galaxies can systematically bias the connection between the mass and the velocity dispersion.

The other systematic effect is the influence of the line of sight (foreground and background) contamination. The problem was recognized a long time ago (Bar-Kana, 1996) with the heuristic suggestion that adding an external shear to an elliptic lens model greatly improves the fits of multiple image configurations. High redshift sources are advantageous from the point of view of dark energy studies and at the same time they are challenging from the point of view of line of sight contamination. One of the most recent studies (Jaroszynski & Kostrzewa-Rutkowska, 2014) trying to quantify the influence of the matter along the line of sight on strong lensing used the technique of simulations of many multiple image configurations, using a realistic model of light propagation in an inhomogeneous Universe model (based on the Millenium simulation). Further progress in this direction has recently been achieved by Collett et al. (2013) in a paper accompanied with publicly available code `Pangloss`. They used a simple halo model prescription for reconstructing the mass along a line of sight up to intermediate redshifts and calibrated their procedure with ray-tracing through the Millenium Simulation.

3.3 Cosmography with cluster lensing

Besides the galaxies acting as lenses, their clusters — the first virialized structures in the Universe — do the same. The cores of galaxy clusters have surface densities which are typically much larger than the critical surface density Σ_{cr} for multiple image production. Therefore they are able to produce strongly lensed images of galaxies and quasars lying behind them. Such images manifest themselves as luminous arcs around clusters. Historically it was Paczynski (1987) who proposed that giant arcs might be gravitationally lensed images of background galaxies. The first measurements of the arcs' redshifts proved this definitively. The possibility of constraining cosmology with CSL systems has been explored in the past e.g. Paczynski & Gorski (1981); Sereno (2002), and still remains a fruitful, fast developing field of research. It is typical that we observe multiple sets of arcs in cluster lenses corresponding to different sources (with different redshifts) lensed by the same cluster. Hence, the abundance of arcs may provide useful cosmological constraints, in a manner similar to the statistics of multiple images in galaxy lenses.

Analogously to the method outlined above in the context of galaxy lensing, the locations of images in cluster lensing systems also contain useful cosmological information. Namely, the image positions depend not only on the mass distribution, but also on the angular diameter distances between the observer, lens, and source. If more than one set of images is observed, the geometrical dependence may be exploited to probe the cosmological parameters even with a single cluster lens. One of the best studied cluster lensing system is Abell 1689. The mean redshift of this cluster is $z_l = 0.184$ and it is one of the richest clusters in terms of the number density of galaxies in its core. Jullo et al. (2010) used this cluster to derive constraints on the cosmological parameters Ω_m and w. Based on images from the Advanced Camera for Surveys (ACS) this cluster is known to produce 114 multiple images from 34 unique background galaxies. This allowed Jullo et al. (2010) to use many observables like (9) from a single cluster. To be more specific instead using $\mathcal{D}^{th}$ like in (9), they used quotients formed pairwise for background sources

$$\mathcal{D}^{th}_{cl} = \frac{\mathcal{D}^{th}(z_l, z_{s1}; \mathbf{p})}{\mathcal{D}^{th}(z_l, z_{s2}; \mathbf{p})} \tag{15}$$

where: z_l is the cluster's redshift, z_{s1} and z_{s2} are redshifts of each respective pair of sources.

By demanding good spectroscopic data for the images and excluding regions where mass reconstruction gets poorer, from the initial 114 images, Jullo et al. (2010) finally selected 28 images which they then used to constrain cosmological parameters to $\Omega_m = 0.25 \pm 0.05$ $w = -0.97 \pm 0.07$.

Even more promising is the idea of using a larger sample of cluster lensing systems. Such an approach has the advantage that results obtained from different lines of sight are statistically independent. As discussed by Gilmore & Natarajan (2009) competitive constraints can be obtained by combining at least 10 lenses with 5 or more image systems. We may therefore conclude that cluster strong lensing is becoming a very useful complementary tool, particularly in probing dynamic dark energy models, which demand larger range of redshifts to be probed.

4 Concluding remarks.

Since the discovery of the first strong gravitational lens system Q0957+561, strong lensing has developed into an important astrophysical tool suitable for investigating both background cosmology and the structure and evolution of galaxies. In this paper, we have presented only few chosen issues: the time delay cosmology and the idea of using Einstein radius measurements of strong lensing systems combined with spectroscopic data (stellar velocity dispersions).

Another important idea not discussed above concerns the studies of dark matter (DM) in galaxies. One of the problems we are facing now is that, according to numerical simulations of large scale structure formation, in the standard cosmology DM halos host a hierarchy of sub-halos, also known as DM substructure. This substructure can hardly be seen in reality. Discoveries of low surface-brightness satellite galaxies around big galaxies like our Milky Way has alleviated this tension a little bit but not to a sufficient degree. One possible explanation is that substructure exists, but it is dark, i.e. subhalos do not form enough stars to be detected. Alternatively, it is possible that subhalos are not as abundant as predicted by numerical simulations. This explanation would imply a major revision of the standard CDM paradigm, either reducing the amplitude of fluctuations on the scales of satellites, or changing the nature of DM from cold to warm. Gravitational lensing provides a unique insight into this problem, since it is the only way to detect dark substructure and measure its mass function. The easiest way to detect te lensing effect of substructure is the perturbation of the magnification pattern. For point sources, the presence of substructure results in ratios of the fluxes of multiple images that are significantly different than what would be predicted by a smooth macro model. This effect is often referred to as the anomalous flux-ratios phenomenon, and has been used to infer the presence of substructure in lens galaxies.

The forthcoming new generation of sky surveys like the EUCLID mission, Pan-STARRS, LSST, JDEM, are estimated to discover from thousands to tens of thousands of strong lensing systems. This clearly heralds a bright future for strong gravitational lensing as a tool in cosmology.

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