Some Notes on the Modelling of Blazhko Effect

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Although hydrodynamic models provide a good fit to most of the nonlinear phenomena in classical variable stars, no reliable models have existed for a long time for the Blazhko effect, a well-known property of RR Lyrae pulsations. During the last decades, several new models have been proposed to understand the puzzle of the Blazhko effect. Some of them are just mathematical considerations to provide a simple mechanism of modulations, lacking a real physical interpretation. Other models are based on results from numerical hydrodynamic calculations.

In parallel to the theoretical attempts to understand Blazhko modulation, space- and ground-based observations of RR Lyrae stars have uncovered a new level of complexity in the pulsation of these stars – new pieces of the great puzzle of RR Lyrae variability have been found. One of the most important recent findings is the limited modulation in the infrared observations. In connection, the radius of the inner layers oscillates in a very regular fashion, lacking significant modulations. It provides a strong physical constraint on all ideas. A satisfactory model has to provide a mechanism not only for the modulations but for all the dynamical effects found in the observations. We provide a reality check based on the observational facts on the possible theoretical models.

Although hydrodynamic models still contain lots of simplifications (1D approximation, lack of non-radial modes in nonlinear calculations, a crude approximation of turbulent convection) they display a very rich behaviour. High order resonances play a crucial role in the nonlinear dynamics representing the interacting modes. The richness of the found phenomena suggests that the interaction of multiple modes should be taken seriously in the understanding of observed features.

1 Introduction

The main properties of classical pulsators (Cepheids and RR Lyrae stars) are well characterised by nonlinear hydrodynamic models. There are several success stories which have already proved that numerical modelling is a primary road leading to the understanding of stellar pulsations (see e.g. Buchler, 2009). Such examples of success stories are the understanding of bump progression in both fundamental and first overtone mode Cepheids. The key to understanding lots of characteristic signatures of nonlinear stellar pulsation is the presence of resonances among/between pulsation modes. That is true for bump progression as well. In general, the nonlinear interaction of pulsation modes plays an important role in sculpturing the oscillation cycles both for the shape and for the modal content. In contrast to the success stories mentioned above, no hydrodynamic models of RR Lyrae stars fit exactly the

Blazhko modulation. However, Smolec & Moskalik (2012) found modulations in BL Herculis models – demonstrating that indeed hydrodynamic models are capable of producing variations resembling the Blazhko effect.

There are several observational constraints that should be satisfied with any successful model of the phenomena. For some general summary, we refer to Kovács (2016) and Smolec (2016) and references therein. A recent and very important addition to the observational constraints has been deduced from infrared observations: Jurcsik et al. (2018) demonstrated that "The radius variation of the photospheric regions associated with the pulsation does not show significant changes in Blazhko stars." They also showed that the modulation of the temperature variations is responsible for the observed modulation of the light curve. This empirical fact alone drastically reduces the possible mechanism that can be responsible for the modulation. All the models, which are not compatible with the inner pulsation of the star with clockwork precision, are ruled out. That is the first observational evidence that the Blazhko phenomenon is strongly related to the top of the atmospheric layers of the stars.

Based on the supposed mechanism, there are different classes of ideas intended to model the Blazhko effect. The three main groups are the following:

- Geometrical effects and/or additional mechanisms e.g. magnetism or circumstellar dust
- Beating of pulsation modes
- Intrinsic dynamical effects

The first attempts to model Blazhko modulation were based on geometrical effects. The simplest idea is the radial pulsation of a rotationally deformed star. Projection effects provide observable modulation only if the rotation and pulsation symmetry axes are oblique. Then the magnetic field should play an important role. These models are mathematically simple, which is their deficiency in the explanation of complex effects. Another simple and well-known mechanism is the beating of oscillation modes – it can elucidate regular modulations. However, pulsation modes with the right amplitude questions this explanation unless some unrealistic physical effects are involved. Due to the complex phenomena associated with Blazhko effect, a mechanism that naturally results in such convoluted process is needed. Then dynamical systems provide an additional candidate for modelling the Blazhko phenomenon. Resonance between radial modes as a possible mechanism for pulsation was first published by Moskalik (1986). By the end of the XXth century, the main concepts of Blazkho modulation reached a physically established form. Shibahashi (2000) presented the theory of oblique pulsator model in its evolved state. In the same conference van Hoolst (2000) showed the nonradial resonant model. It is noteworthy that Robert Buchler suggested a new idea during the discussion of the lecture: *"Furthermore, these modes (which we labeled strange modes) can be in 4:1 resonance. This is a promising scenario in the Blazhko effect and we are going to check it with hydrodynamic modelling."* Although this exact assumption failed, an altered variant with 9:2 resonance survived.

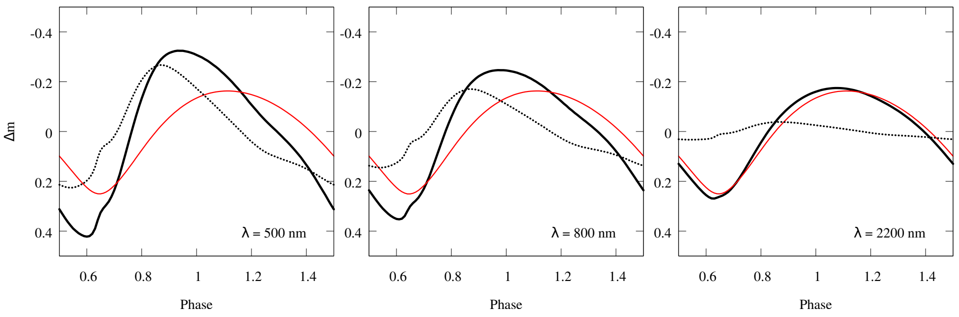

Fig. 1: Light variation of an RR Lyrae model for three different wavelength band (λ_c =500, 800 and 2200 nm. The total variation, Δm_R and Δm_T are displayed by solid thick, solid thin, and dotted curves.

2 What does Hydrodynamics Tell Us?

Since a very important new observational constraint restricts the radius variation of the photospheric region, it is necessary to check whether the numerical models predict such effects. It is also quite important to check the identified dependencies on the observational wavelength bands. Therefore we performed some numerical calculations to predict the behaviour of some models on the radius modulation.

Since the Budapest-Florida code handles radiation transfer in the diffusion limit with the grey approximation, there are no direct data in monochromatic radiation or in photometric bands. However, based on the temperature and density profile at a given phase of the pulsation, it is possible to estimate the optical depth in different wavelength ranges through the profile. In order to estimate the effects of colours, we used the monochromatic opacity tables by the Opacity Project (Seaton et al., 1994). The opacity was averaged with a Gaussian window with a half-width of 100 nm, and the colour dependent optical depth (τ_λ) of the model zones were calculated for the whole optical band. Then based on the physical quantities of the model shells, the temperature and radius profiles could be calculated as a function of optical depths. As a result, the radius and temperature variation for a fixed τ_λ can be obtained. The $R(\tau_\lambda, t)$ and $T(\tau_\lambda, t)$ profiles then can be used to predict the light variation in different wavelength bands.

We used the simplest estimate, the Eddington-Barbier approximation, to predict the range of light modulations. The star's emergent flux is proportional to the source function ($S_\lambda(T)$) at τ_λ =2/3. Then it is possible to approximate the contribution of the temperature and the radius change to the light curve:

$$\Delta m_{T,\lambda} = \quad -2.5 log(S_\lambda(\hat{T}_\lambda)/S_\lambda(\hat{T}_0)) \tag{1}$$

$$\Delta m_{R,\lambda} = \quad -5.0 log(\hat{R}_\lambda/\hat{R}_0), \tag{2}$$

where $\hat{T}_\lambda$ and $\hat{R}_\lambda$ is the temperature and the radius at τ_λ=2/3.

We approximate the source function by the Planck-curve. The observable light variation in this approximation is $\Delta m_\lambda = \Delta m_{T,\lambda} + \Delta m_{R,\lambda}$. Figure 1 displays the calculated variations for three different lambda values (giving a close estimate to the

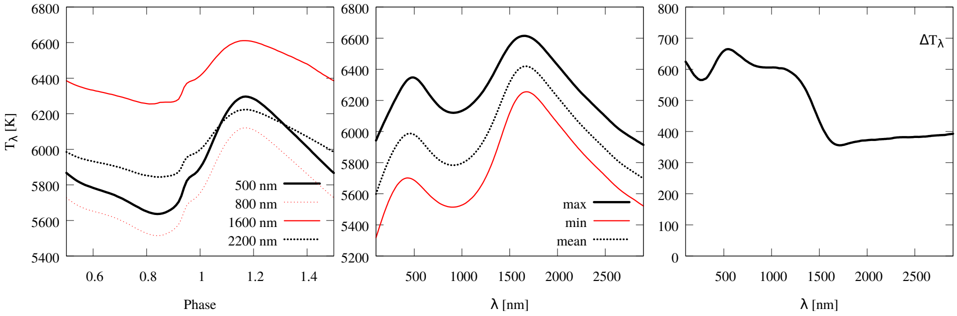

Fig. 2: Temperature variation of the layer $\tau_\lambda = 2/3$.

V, I, and R light curves). The model parameters are the following: $M = 0.60\,\mathrm{M}_\odot$, $L = 45\,\mathrm{L}_\odot$, $T_\mathrm{eff} = 6000\,\mathrm{K}$, $X = 0.75$, and $Z = 0.001$. In agreement with the observational results (Jurcsik et al., 2018), in the visible band the light variation is strongly affected by the changes due to the temperature variation and in the infrared band, the radius variation dominates the light curve. As expected, the amplitude of the radius variation does not depend on the photospheric optical depth in the range of the optical continuum radiation. However, the temperature has a strong gradient in the atmosphere, resulting in a more pronounced effect on the wavelength dependence of the light curve. Contrarily to the usual interpretation, we do not see deeper into the atmosphere in infrared than in the visible bands in all phases of the pulsations. To demonstrate this effect, we display the characteristics of the temperature variation $\hat{T}_\lambda(t)$ on Figure 2. The left box shows the temperature curves for different wavelengths. Due to the opacity minimum around 1600 nm, we see deepest into the atmosphere at that wavelength. Around the K band, the optical depth is comparable to the visual range. However, at the temperature maximum, the longer wavelength gives a lower temperature, i.e. a shallower region in the atmosphere compared to the shorter wavelengths. That is the main reason for the reduced weight of temperature in the K light curve. The middle panel shows the dependence of the temperature extrema and mean value on the wavelength, while the minimum to maximum amplitude is presented in the right panel. It further demonstrates, that the temperature effect is the highest at the visible band, and it is significantly decreased in the 2000 nm range.

The observational constraints restrict what degree the radius variation contributes to the modulations. Thus, it is critical to investigate how the various ideas manifest in this relation. We cannot test each model in this manner, but the beat, the convective modulation, and the period doubling mechanism make it possible to predict the radius effect with their assumptions.

We tested the beat mechanism on a double mode RR Lyrae model with the following parameters: $M = 0.82\,\mathrm{M}_\odot$, $L = 40\,\mathrm{L}_\odot$, $T_\mathrm{eff} = 6300\,\mathrm{K}$, $X = 0.75$, and $Z = 0.001$. This model is close to a 3:4 resonance, and both the fundamental mode and the first overtone play a role in the pulsation. The top boxes of Figure 3 show the beating effect on the fundamental mode light variation, the temperature and radius change determined light curves. As expected, the modulation is comparable in all three magnitude curves. Although this double mode model does not represent exactly the one in the corresponding idea (Bryant, 2014), the result indicates that the beating model is not compatible with the observations.

The effect of Stothers' idea (Stothers, 2006) is investigated by running the hydrodynamic calculation with the following parameters: $M = 0.59\,\mathrm{M}_\odot$, $L = 56\,\mathrm{L}_\odot$, $T_\mathrm{eff} = 6500\,\mathrm{K}$, $X = 0.75$, and $Z = 0.001$, with different mixing-length parameters (for such tests see also Molnár et al., 2012). The amplitude of the pulsation depends on the efficiency of turbulent convection. The results are displayed in the middle row of Figure 3. Again, the modulation present in both the radius and the temperature contribution to the light curve, but the effect is less prominent in the radius curve. The stronger dominance of the temperature effect on the modulations shows that there are natural processes that differentiate between radius and temperature effects.

The hydrodynamic results that show cycle-to-cycle variations in RR Lyrae models are the ones with period doubling. Since some important properties of period doubled RR Lyrae models have not been published yet, here we summarise some preliminary results we had. As it was demonstrated by Kolláth et al. (2011) the mechanism behind the period doubling is the resonant interaction of the fundamental mode with the 9th overtone. Buchler & Kolláth (2011) found that the same resonance can result in amplitude modulation resembling the Blazhko effect. Therefore, the investigation of the nature of period doubling can help in understanding dynamical effects with a possible relation to modulations in real stars. The anomaly of the modulation in infrared again raises the question of how period doubling manifests itself in different observable quantities. As for an example, we randomly selected an RR Lyrae model with period doubling from our model sequences.

The lower panel of Figure 3 presents the results obtained for the period doubled model (the same parameters as above, but with the standard mixing length parameter). Remarkably, the radius variation gives only a negligible contribution to the total light curve modulation, the cycle-to-cycle variation of $\hat{R}_\lambda$ is almost negligible, compared to the temperature variation induced changes. This result is in close relation with the fact that the 9th overtone involved in the process is a strange mode which has a surface dominated eigenfunction. In addition, according to our calculations, the relative strength of the period doubling is significantly reduced in the K wavelength range compared to the visual band. These results altogether provide a perfect fit to the findings of Jurcsik et al. (2018). Moreover, the lower pulsation amplitude and period doubling strength in the K band should indicate a less dramatic modulation in the K light curve.

3 Models Versus Constraints

The possible models are constrained by observational facts and also by the laws of physics. Even if a model can reproduce all the observed features of the phenomenon, it is "disqualified" if it contradicts the basic laws of physics or the proved properties of stellar pulsation theory.

The major observational constraints that one should take into consideration are the following:

- **The amplitude of the modulation (MoAm)** is comparable to the amplitude of the pulsation mode itself. This is one of the toughest constraints a

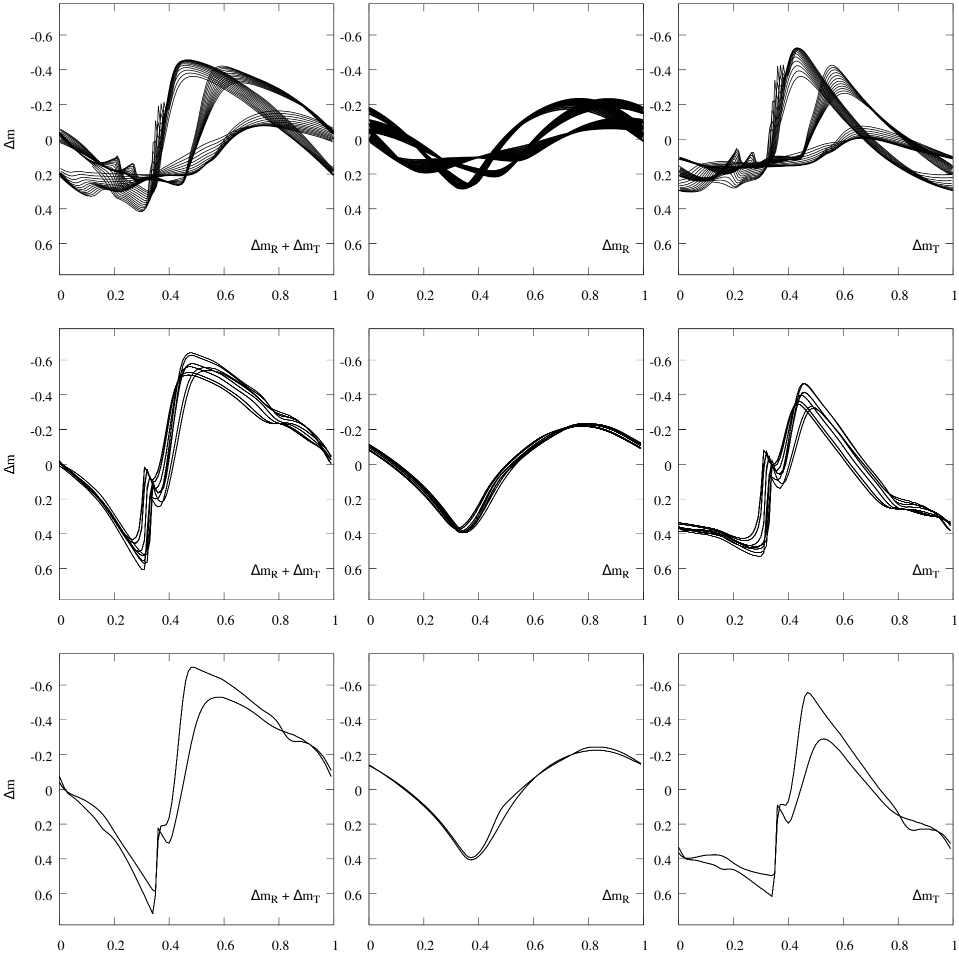

Fig. 3: The contribution of the temperature and radius induced variations to the light curve. Top: beating model, Middle: variable mixing length, Bottom: in a period doubled model.

Table 1: Models vs. observational constraints.

Model	MoAm	InRa	TiSc	SiLo	AdMo	IrAn	PeDu	Phys
Oblique rotator	?	Yes	Yes	No	No	Yes	No	Yes
Turb. modulation	No	Yes	Yes	Yes	No	No	No	Yes
Atm. shocks	?	?	?	No	No	Yes	No	No
F/o1 beating	No	No	Yes	No	Yes	No	No	No
F-NR beating	No	?	Yes	No	Yes	No	?	Yes
F-NR resonance	?	?	Yes	Yes	Yes	?	?	Yes
9:2 resonance	?	?	Yes	Yes	Yes	Yes	Yes	Yes

model has to predict. Most of the mechanisms are weak in generating large amplitude modulations.

- **The incidence rate (InRa)** of modulated stars are high, about 50% (see e.g. Jurcsik et al., 2009; Prudil & Skarka, 2017). The models should also answer what is the selection process between modulated and stable pulsations.

- **The time scale of the modulation (TiSc)** is also an important empirical parameter of the Blazhko effect – all modulation scales can be handled by the theory. In addition, multiple modulation scales present another restriction for most of the models.

- **Structure of the side lobes (SiLo)**, asymmetries, complex frequency displacements excludes lots of the mechanism from the acceptable list.

- **Extra (Additional) modes (AdMo)** or peaks in the Fourier spectra can be a feature independent from the Blazhko effect, but the pulsation models should provide some clues on the multi-mode properties of RR Lyrae stars.

- **Anomaly of the infrared and the radius modulation strength (IrAn)** is a very recent constraint (Jurcsik & Hajdu, 2017; Jurcsik et al., 2018) on theory. It is also a dichotomy between radius variation and the light curve or it can be simply treated as the dependence of modulation amplitude on the depth of the stellar atmosphere.

- The occurrence of **period doubling (PeDu)** in Blazhko modulated light curves present another possible constraint on models. Note that it appears with high amplitude only at specific phases of the Blazhko cycle.

- Finally any idea that is not compatible with the known physics of stellar pulsation should be ruled out (**Phys**).

The quality of models related to the constraints is summarised in Table 1. In some cases, it is not possible to make a decision, mostly because of a lack of numerical hydrodynamic modelling. For example, the nonradial resonance hypothesis cannot be tested in real models since no 3D models are available to check the nonlinear properties of nonradial pulsations and also the turbulent convection is handled only in crude approximations.

The oblique rotator model was the first idea to interpret the modulations, but it fails in several aspects. An additional weakness is the lack of magnetic fields needed for a significant effect. The turbulent modulation (Stothers, 2006) is physically a reliable idea, but it is not compatible with the acceptable variation of strength in turbulent convection (Smolec & Moskalik, 2011; Molnár et al., 2012).

Our main criticism of the atmospheric shock hypothesis is that the dynamical modelling of the shock could not be treated separately from the pulsation dynamics. Shock waves are part of the whole nonlinear hydrodynamic system; they originate in global processes. Therefore one cannot speak about overtone shock (Gillet, 2013) without significant oscillation amplitude in overtone pulsation. Furthermore, shock waves are present in hydrodynamic models, so if the idea is correct it should appear naturally in hydrodynamic models. We have to note, however, that atmospheric shocks significantly shape the observed quantities – for example, the radial velocity curve in the line forming region can be modulated while in the deeper regions of the star, the radius variation does not display significant alterations (Jurcsik & Hajdu, 2017; Jurcsik et al., 2018).

Beating models, in general, cannot result in high modulation amplitude, and the variation misses the observed complexity unless some extra interaction is included. The simplest case would be the effect of radial modes (e.g. fundamental and first overtone) with close frequencies as proposed by Bryant (2014). However, this idea assumes a huge nonlinear period shift that is irreconcilable with reliable physics. That is why this model is disqualified by basic theoretical rules. This model was modified to beating with nonradial modes (Bryant, 2015) – then at least the modulation amplitude is questionable to reach the necessary level. In addition, when some nonlinear effects like resonance are included, then the model is close to the resonant hypothesis. The physically most sound nonradial resonance model is given by Nowakowski & Dziembowski (2001). However, it does not predict the full complexity of the observations. Due to the lack of nonlinear 3D models, the predictions of the idea cannot be checked.

4 Conclusion

The most recent observational constraint by Jurcsik et al. (2018) rules out lots of the models. Similarly, the period doubling observed in some phases of the Blazhko cycle restrict the possible mechanisms. There is no existing model which exactly satisfy all the observational constraints of Blazhko modulation. However, the fundamental to 9th overtone, 9:2 resonance hypothesis at least does not contradict to any of the observational facts. We have to add that during the editorial process of this manuscript the first RR Lyrae models with amplitude modulations have been found. These new results will be published elsewhere.

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