Blazhko Light Curves – Non-Modulated and Harmonic Detuned Signals

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In this talk, I showed that two common assumptions concerning RR Lyrae light curves are invalid. (1) It was demonstrated that the presently best fitting mathematical model of Blazhko stars' light curves is a Fourier representation of a general almost periodic function. It follows that, the widely used phrase 'Blazhko modulation' is at least not precise. Although many features of the Blazhko light curves can be described in the modulation framework, the real light curves are definitely not modulated signals. That is, the external modulation explanations of the Blazhko effect must be wrong, which gives a further support for those theoretical models which produce the effect as an inherent feature of the pulsation. (2) Up to now most studies assumed that the Fourier spectrum of the light curve of an RR Lyrae star, either monoperiodic or showing Blazhko effect, is dominated by the main pulsation frequency and its exact harmonics. The Fourier representation of the almost periodic function, however, contains frequency terms which systematically deviated from the harmonics. The previously published deviation of the harmonic frequencies of V445 Lyr is explained with the harmonic detuning effect predicted by the almost-periodic function framework. Using the original *Kepler* RRab data, an additional star (V2178 Cyg) has also been found showing this detuning effect.

1 Introduction

Since the topic is discussed in detail by a recently published article (Benkő, 2018), here the main results, and some in the meanwhile newly arisen questions, are presented.

Traditionally, there were two main concepts which may explain the detected features of the Blazhko light curves and their Fourier spectra. One of them is a beating phenomenon of two close-by frequencies (Kolenberg et al., 2006). This scenario explains the amplitude variation and the typical Fourier spectra obtained from the ground-based observations but it fails when we want to describe the fine details of space-based observations like multiplet structures in the Fourier spectra. Therefore, this framework is looked on as an out-of-date one (Kolláth, 2018).

The alternate handling is the modulation framework (Benkő et al., 2011) in which we assume a non-modulated light curve as a carrier wave, and the observed features are explained by simultaneous amplitude and frequency modulations of the carrier wave signal. This picture can describe more details than the first case, but it has its own problems. It needs a complicated fitting formula, and the fits are not optimal: when we subtract a fit from the observed data, significant residual remains.

On a purely empirical basis, Szeidl et al. (2012) found a formula which fits well the observed light curves. They did not discuss the mathematical features of their formula, which seems to be a generalised modulation formula but it turned out to be much more.

2 Almost Periodic Functions and Their Application

The empirical formula suggested by Szeidl et al. (2012) is:

$$m(t) = m_0 + \sum_{k=1}^{l} b_k \sin(2\pi k f_{\mathrm{m}} t + \varphi_k^{\mathrm{b}}) + \sum_{i=1}^{n} \left[a_i + g_i^{\mathrm{A}}(t)\right] \sin\left[2\pi i f_0 t + \varphi_i + g_i^{\mathrm{F}}(t)\right]. \tag{1}$$

Here the notations are the usual: $m(t)$ represents the light curve (brightness in the function of time t), f_0 and f_{m} mean the main pulsation and the modulation frequencies, i, k are running integer indices, a, b and φ coefficients are the Fourier amplitudes and phases, respectively; m_0 is the zero point constants; l and n integers show the number of terms in the finite Fourier sums. The modulation functions are

$$g_i^{\mathrm{M}}(t) = \sum_{j=1}^{l_i^{\mathrm{M}}} a_{ij}^{\mathrm{M}} \sin(2\pi j f_{\mathrm{m}} t + \varphi_{ij}^{\mathrm{M}}), \quad \mathrm{M} = \mathrm{A} \text{ or } \mathrm{F}. \tag{2}$$

The upper index A indicates the amplitude modulation and index F means the frequency modulation.

The formula (1) has similar structure as the modulation one (see Benkő et al. 2011) but with two main differences. (i) As opposed to the modulation case, the zero point variation does not contain explicitly the modulation function. (ii) The more surprising difference is the form of the modulation functions: here the modulation functions depend on the index i (the Fourier harmonics of the pulsation signal). That is, we are facing with a 'modulation' that affects each harmonic differently. How does the modulation know about the Fourier solution of the pulsation signal? These harmonics are the mathematical consequences of the non-sinusoidal shape of the carrier wave. They have no direct physical meaning.

I found the solution of this mystery when I met the idea of the almost-periodic function. The almost-periodic functions are a kind of extension of the periodic function set. This function set was defined and investigated first by Harald Bohr in the 1920s (Bohr, 1947).

It can be proven (Benkő, 2018) that both the fitting formula of the modulation handling and the empirical formula in Eq. (1) are Fourier representations of almost-periodic functions. What follows from this? We have two different fitting formulae which are both Fourier representation of almost-periodic functions. The modulation formula describes a general external modulation of the pulsation signal. The empirical formula fits better the observed light curves. At the same time we know that the Fourier representation of almost-periodic functions are unique (Bohr, 1947), and these two functions are different, consequently, the observed light curves can not be modulated signals. In other words, the Blazhko effect is not a simple external

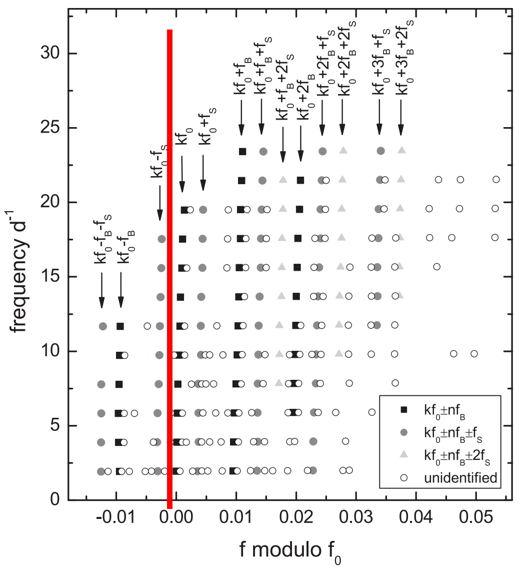

Fig. 1: Echelle type diagram from the detected frequencies of V445 Lyr *Kepler* data (fig. 4 in Guggenberger et al. 2012). The deviations of the harmonic components f_n from the exact harmonic positions nf_0 (shown by the vertical line) are evident. The linear combination frequencies contain f_n also show deviations.

modulation on the pulsation but a more complex feature. This excludes all those physical explanations for the Blazhko effect such as binarity, global magnetic field, etc., which assume external modulation and implicitly supports those that explain Blazhko phenomenon within the pulsation theory (e.g. Buchler & Kolláth 2011).

3 The Harmonic Detuning Effect

There is another important consequence of the almost-periodic handling. If we investigate the instantaneous frequency of the Fourier components, we get the following formula:

$$f_n(t) = nf_0(t) + \frac{1}{2\pi}\varphi'_n(t). \tag{3}$$

Here φ' means the time derivative of the phase variation functions. There is no one-to-one correspondence between the instantaneous frequencies here and the detected frequencies in the Fourier spectra. However, it is true that the average frequency of the Fourier spectrum equals the time average of the instantaneous frequency. It can be seen that the components of the Fourier solution contain not harmonic terms but slightly different ones. This effect is called the harmonic detuning effect (HDE). It is a prediction of the almost-periodic framework, but can this effect be detectable?

Up to now, the experience was that the Fourier spectrum of an RR Lyrae star, either monoperiodic or Blazhko star, is dominated by the main pulsation frequency and its exact harmonics. The only exception was the *Kepler* RR Lyrae star V445 Lyr which showed a very strong and complex Blazhko effect. Guggenberger et al. (2012) constructed the diagram in Fig. 1. Here, in the horizontal axis, is the frequency modulo to the main pulsation frequency while the vertical axis shows the frequency. The position of the main pulsation frequency and its harmonics are connected with a (red) continuous line. A remarkable deviation of the detected frequencies can be realised from this line. This was the reason why Guggenberger et al. (2012) reported significant and systematic deviation of the 'harmonics' from their exact harmonic positions.

Investigating this finding, the deviation parameter was defined as

$$D_n = \left| \frac{f_n}{f_0} - n \right|. \tag{4}$$

It is equivalent with the f_n modulo f_0 used in Fig. 1. For calculating D_n (and their accuracy), we need the frequencies f_0, f_n, and their accuracies, as precisely as possible. For this purpose the following have been done: (i) The tailor-made photometric data of the *Kepler* Blazhko stars from Benkő et al. (2014) and a similar data set prepared for non-Blazhko stars were collected. (ii) A frequency analysis has been done using SigSpec (Reegen, 2011) program package. This tool gives not just frequencies but their spectral significancies as well. The frequency determination accuracy was calculated with the help of these spectral significancy values. This estimation gives smaller and a more realistic error for frequency determination (see Kallinger et al. 2008) than the traditional Rayleigh frequency resolution. (iii) Raw frequency spectra without any pre-whitening were always used because each pre-whitening step could affect the residual spectrum. In practice, the SigSpec was run once for getting the dominant frequency, and then again and again on smaller intervals around each harmonic for calculating the higher order components. This process provides that we always work on raw spectra.

The investigated sample contained 17 Blazhko and 19 non-Blazhko stars of the original *Kepler* field. Significant HDE was found for two stars(Benkő, 2018). In the case of V445 Lyr, the formerly published deviation of the harmonic components from their exact harmonic positions (Guggenberger et al., 2012) was verified and explained with HDE. The case of V2178 Cyg is an additional example star for showing HDE.

We investigated 17 Blazhko stars and found HDE for two stars only. Why? The almost-periodic handling always predicts HDE. What does affect the detectability of HDE? First, we need for precise frequencies long-baseline and accurate light curves, otherwise the frequency deviations are within the error bars of the exact harmonic positions. This condition is fulfilled for all *Kepler* RR Lyrae stars. What else do we need? We can re-write the deviation parameter of Eq. 4 in an alternate form:

$$D_n = \left| \frac{\langle \varphi_n' \rangle}{2\pi f_0} \right|. \tag{5}$$

From this expression we see that the determining quantity is the time average of the derivative of the phase variation function $\langle \varphi_n' \rangle$. This value could be large enough if the phase variation function has large enough amplitude and/or asymmetric (irregular) behaviour. Otherwise, since the phase variation is happening with the Blazhko period, a small amplitude phase variation resulted in a small amplitude derivative changes as well, which could be undetectable. In the case of a symmetric regular phase variation, the time average of φ_n and its derivatives became zero. That is, the effect averages out.

Since the total phase variation can be typified by the well-known O−C diagrams as well, we do not need to prepare the phase variation curves. For a first guess, it is enough to check the O−C diagrams. It can be illustrated with figure 7 in Benkő et al. (2014) where the O−C diagrams of the *Kepler* Blazhko stars are presented. The two highest amplitude and most asymmetric curves are the V445 Lyr and V2178 Cyg. We found HDE for those two stars. This figure also indicates a possibility when the phase variation function is essentially symmetric but its observed part is not. This could happen if the Blazhko period is long and the observed time span covers it just partially (see the case V354 Lyr in fig. 7 of Benkő et al. 2014). Such a situation can result in an asymmetry in the observed phase variation and also detectable HDE. Similar phenomenon could be caused by a high rate period change as well, which is frequent among Blazhko stars.

As a summary: be cautious with the Fourier spectra of Blazhko stars! You should never assume the higher order components as harmonics before you checked their actual positions carefully.

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