Uncertainties in Evolutionary Models of Classical Pulsators

Maurizio Salaris[1]

1. Astrophysics Research Institute, Liverpool John Moores University
146 Brownlow Hill, Liverpool L3 5RF, UK

RR Lyrae and Cepheid stars have played and still play an important role in the determination of the cosmic distance scale. Although their evolutionary status is well determined, stellar evolution models for these stars are still affected by a number of uncertainties that affect the predictive power of theory. This review addresses the main open problems in modelling the evolution of these classical pulsators.

1 Introduction

Radial pulsators like RR Lyrae (RRL) and classical Cepheids have played and still play a major role in the determination of the Galactic and extragalactic distance scale, and the value of the Hubble constant (see, e.g., Beaton et al., 2016; Riess et al., 2016). It is well established that, from an evolutionary point of view, RRL stars are low-mass core He-burning objects (horizontal branch – HB – stars) crossing the instability strip, while classical Cepheids are their intermediate–high-mass counterpart.

Even though we know very well the evolutionary stage and general mass range of these two classes of classical pulsators, we are still unable to theoretically predict accurately mass and luminosity distribution of RRL and/or Cepheid stars in a stellar population for a given star formation history. In the case of RRL pulsators, problems arise from the lack of a predictive theory for the efficiency of surface mass loss in red giant branch (RGB) low-mass stars, and the uncertain treatment of mixing in the cores of He-burning models. Regarding classical Cepheids, uncertainties in core-mixing during both H- and He-burning, and the modelling of rotational mixings are the main culprits.

The following sections will present a brief analysis of these problems, the solutions adopted in stellar evolution modelling, and recent developments.

2 RGB Giant Mass Loss

Mass loss (ML) affects all stages of stellar evolution and its parametrization remains a major problem in stellar modelling. Satisfactory empirical determinations as well as a self-consistent parameter-free physical descriptions are still lacking. Regarding RGB stars, the unknown ML law prevents a first-principle prediction of whether an old population with a given star formation history hosts an RRL population, and eventually the mass, $T_{\rm eff}$, luminosity, and period distributions of the pulsators.

The classical Reimers formula (Reimers, 1975) is still widely used in RGB modelling, after tuning in some way the Reimers free parameter, η. Other formulations do exist, as summarized by Catelan (2009), all requiring calibrations of free parameters. Concerning RGB metal poor stars, a traditional way to tune the ML employing globular clusters was to reproduce the observed HB morphologies in Galactic globulars with synthetic HBs, by tuning the Reimers free parameters (and an eventual spread around the mean η value) for a given cluster age and chemical composition. Nowadays, we know that this procedure is risky because of the presence of He-abundance spreads amongst stars in individual clusters, which add an additional free parameter to tune when producing synthetic HBs (see, e.g. Dalessandro et al., 2011).

Recent semiempirical estimates of ML rates in RGB stars are based on observations of the infrared excess due to circumstellar envelopes around mass-losing RGB stars (Origlia et al., 2014). By modelling the emerging spectrum and dust emission at the observed wavelengths (that entails a number of assumptions, like the gas-to-dust ratio, dust composition, density profile of the expanding envelope) and integrating the resulting ML rates over the theoretical lifetimes of the observed stars, Origlia et al. (2014) derived from a sample of 15 globular clusters:

$$\Delta M_{\rm RGB}({\rm M}_{\odot}) = 0.08[{\rm Fe/H}] + 0.24 \pm 0.03,$$

where $\Delta M_{\rm RGB}$ is the total amount of mass lost along the RGB by a low-mass star with a given [Fe/H].

3 Core Mixing in He-Burning Stars

In a low-mass star after the He-flash, He-burning is efficient in a convective He-core, and the opacity is dominated by electron scattering with an important contribution (about 25% of the total) from *free-free* absorption. The transmutation of He to C due to nuclear burning increases the *free-free* opacity, and within the convective core the radiative temperature gradient, $\nabla_{\rm rad}$ ($\nabla_{\rm rad} \equiv {\rm dlog}(T)/{\rm dlog}(P)$), increases, developing a discontinuity at the inner convective boundary (see Fig. 1).

This discontinuity of the radiative gradient is unphysical. Given that the convective flux, $F_{\rm c} \sim (\nabla_{\rm rad} - \nabla_{\rm ad})$, there would be an increasing convective flux developing on the inner side of the convective boundary, whilst this would be formally zero on the radiative side of the boundary. According to Gabriel et al. (2014), the presence of this discontinuity is due to an incorrect application of the Schwarzschild criterion when a mixing-length picture of convection is considered, and a proper implementation requires always $\nabla_{\rm rad} = \nabla_{\rm ad}$ at the convective side of the boundary. In practice, at every computational timestep, one can place the boundary of the fully mixed convective region at the layer where $\nabla_{\rm rad} = \nabla_{\rm ad}$ after mixing.

After this early phase, when the extension of the convective core follows the increase of $\nabla_{\rm rad}$, the radiative gradient profile starts to show a minimum (profile 3 in Fig. 2), as a consequence of the progressive outwards shift of the convective boundary. This minimum arises from the complex behaviour of the physical quantities involved in the definition of $\nabla_{\rm rad}$, such as opacity, pressure, temperature, and local energy flux. Now an outward mixing to eliminate the $\nabla_{\rm rad}$ discontinuity will induce a general decrease of the radiative gradient in the whole convective core (due to an average increase of He and consequent decrease of C – see profile 4 in Fig. 2).

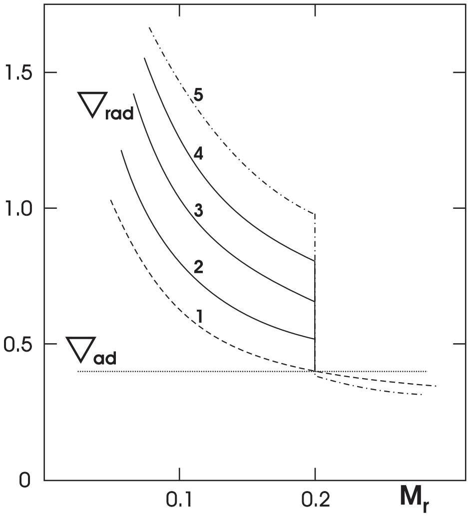

Fig. 1: Time evolution of the radiative temperature gradient $\nabla_{\rm rad}$ as a function of the mass enclosed within radius r inside a core He-burning model, if the gradient discontinuity is maintained. Increasing numbers define a sequence of increasing time.

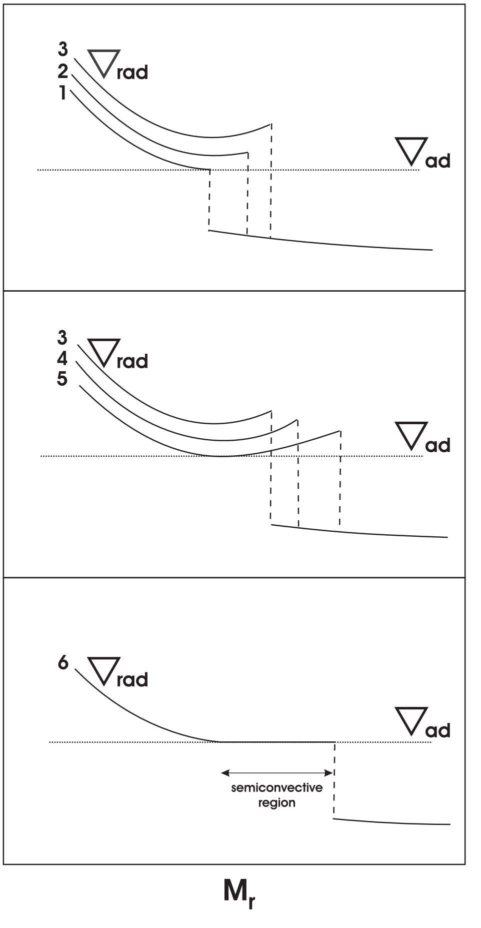

Fig. 2: Similar to Fig. 1, a sketch of the time evolution of the radiative gradient profile near the boundary of the convective core during core He-burning stage. The panels show the sequence of events which lead to the formation of the semiconvective zone (shown in the bottom panel). Increasing numbers denote a sequence of increasing time

The radiative gradient decreases and becomes equal to $\nabla_{\rm ad}$ at the location of the minimum of $\nabla_{\rm ad}$ (profile 5 in Fig. 2).

The region between the 'neutral' $\nabla_{\rm rad} = \nabla_{\rm ad}$ point, and the overlying formally convective shell whose upper boundary still displays a discontinuity in $\nabla_{\rm rad}$, cannot be instantaneously fully mixed, otherwise the minimum of $\nabla_{\rm rad}$ would drop below the adiabatic value, and hence a portion of the mixed region would no longer be convective. A stable, somewhat *ad-hoc* solution to this problem (see, e.g., Castellani et al., 1985, and references therein) is to impose a partial mixing (denoted as 'semiconvection') in the formally convective shell such that the final chemical composition – shown in Fig. 3 – satisfies the condition $\nabla_{\rm rad} = \nabla_{\rm ad}$ (profile 6 in Fig. 2). The mass location of the minimum of the radiative gradient moves outwards with time because of the evolution of the chemical abundances caused by nuclear burning. As a final result, the C-enriched region increases its size outwards.

The effects of this *semiconvection* on the evolution of low-mass core He-burning stars are the following: The evolutionary tracks perform more extended loops in the Hertzsprung-Russell diagram – thus affecting the predicted mass of RR Lyrae stars – and the central He-burning phase lasts longer because of a larger amount of fuel to burn.

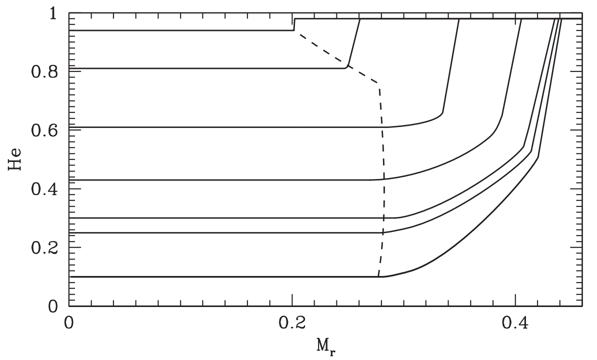

Fig. 3: Helium abundance profile inside the convective core and the semiconvective zone at various levels of central He depletion, for a low-mass core He-burning model. The dashed line marks the location of the fully mixed core boundary.

When the central He-abundance has decreased to about $Y \sim 0.10$, He-burning becomes mainly a $^{12}\mathrm{C} + \alpha$ production of oxygen, whose opacity is even larger than that of ^{12}C. This causes an increase of the size of the semiconvective region and, in turn, more fresh helium is transferred into the nearly He-depleted core. Even small amounts of He added to the mixed core enhance the rate of energy production; the luminosity increases, driving an increase of the radiative gradient. As a consequence, a phase of an enlarged mixed zone starts, the so-called *breathing pulse* (BP). After this BP, the star readjusts to burn steadily in the core the fresh He driven there by semiconvection. A few breathing pulses are expected before the complete exhaustion of He in the core. The evolutionary effects of the breathing pulses are the following: The models perform a loop in the Hertzsprung-Russell diagram at each pulse, the He-burning lifetime is slightly increased, and the mass of the CO-core at He exhaustion is increased.

Empirical constraints – mainly the number ratio between HB and asymptotic giant branch stars in Galactic globular clusters, the so-called R_2 parameter – suggest that the efficiency of the BPs is very low, if any (Caputo et al., 1989; Cassisi et al., 2003). Therefore they are usually inhibited in stellar model computations by using *ad hoc* numerical assumptions (Dorman & Rood, 1993; Cassisi et al., 1999). Typically, during the late stages of core He-burning, one forces *by hand* the extension of the mixed region to prevent an increase of the central He abundance from one model to the next one (Caputo et al., 1989). Another option is to set to zero the gravitational term in the energy generation equation (clearly an unphysical ad-hoc solution) for the inner regions. In this way, the BPs are also effectively inhibited (Dorman & Rood, 1993).

This brief discussion highlights the difficulty in modelling core mixing in HB stars. The standard treatment is usually instantaneous mixing, that may not be adequate in the semiconvective layers. It is also worth mentioning that with the inclusion of an extended overshooting region ($\sim 1\,H_P$, where H_P is the local pressure scale height) beyond the layer where the $\nabla_{\rm rad}$ discontinuity develops, the fully mixed core is always so large that the need to include semiconvective layers disappears (Straniero et al., 2003, but breathing pulses still seem to appear also with large overshooting). An important consequence of including semiconvection, or large overshooting and/or breathing pulses, is that the CO profile in the final CO core changes, with important consequences for the cooling times of the final white dwarf stage.

With increasing stellar mass the weight of the *free-free* opacity decreases (because of higher core temperatures) and eventually all these problems disappear in the regime of high mass stars (masses above $\sim 10\,\mathrm{M}_\odot$).

Very recently, the asymptotic gravity-mode period spacing ($\Delta\Pi_1$) of core He-burning stars in the *Kepler* field has been employed as powerful diagnostic of core mixing in a sample of field stars with masses between ~ 1 and $\sim 2.0\,\mathrm{M}_\odot$, and stars in NGC6819 and NGC6791 open clusters, by Constantino et al. (2015), Constantino et al. (2016), Bossini et al. (2015), Bossini et al. (2017). These analyses have shown, first of all, that an increase of the size of the mixed region beyond the discontinuity of the radiative gradient is required, in agreement with theoretical arguments. Also, the semiconvective treatment sketched before gives a decent match to the asteroseismic data, although two different types of mixing are actually favoured. Bossini et al. (2017) show that the asteroseismic data are well matched if mixing beyond the 'neutral' $\nabla_{\rm rad} = \nabla_{\rm ad}$ layer is treated as instantaneous and extended by $0.5H_P$ beyond this neutral point. On the other hand, Constantino et al. (2015) best match the asteroseismic data with the so-called *maximal-overshoot scheme*. In this formalism, the core is then allowed to grow only if $\nabla_{\rm rad}/\nabla_{\rm ad} > 1 + \delta$ everywhere in the mixed region, where δ is a (small) free parameter. This ensures at all times full mixing between the minimum of the radiative gradient and the overlying formally convective shell. In a way, both these asteroseismic-based mixing schemes tend to maximize the fully mixed region compared to the standard semiconvective treatment.

Regarding BPs, the theoretical analysis by Spruit (2015) has given very likely a theoretical justification for the suppression of this phase. In a nutshell, due to the strong buoyancy of He entering a mixed region with a C+O+He mixture, only a small amount of additional He can be carried down in the convective flows, inducing a limit to the rate at which He can be ingested into the core. A formula for the maximum ingestion rate is provided, and has been applied very recently to core He-burning models by Constantino et al. (2017). These authors have shown that BPs are automatically suppressed when this condition is implemented.

4 Convective Boundary Mixing and Rotational Mixing

In real stars mixing beyond the formal Schwarzschild (or Ledoux) boundary is expected to arise from the interplay of several physical processes (Andrássy & Spruit, 2013, 2015; Viallet et al., 2015), grouped in stellar evolution modelling under the term *overshooting* or, better, *convective boundary mixing* (CBM). Quantifying the efficiency of this CBM is uncertain and involves free parameters. The standard approach is to assume that CBM into the formally stable layers does not affect their thermal structure, and hence the temperature gradient stays radiative. The composition is then mixed instantaneously between the formal convective boundary, and layers at a distance λH_P from this border, where λ ($\lambda < 1$) is a free parameter to be observationally calibrated, and H_P is the pressure scale height at the convective border. Another formalism (Herwig, 2000) describes CBM as a diffusive process (avoiding the instantaneous mixing approximation) based on results of 2D radiation-hydrodynamics simulations of shallow stellar surface convection zones (Freytag et al., 1996). The diffusion coefficient $D_{\rm ov}$ is given by

$$D_{\rm CBM} = D_{\rm c} \exp\left(-\frac{2z}{f H_P}\right),$$

where $D_{\rm c}$ is the diffusion coefficient inside the convective region (approximated as $D_{\rm c} = (1/3)\alpha_{\rm MLT} v_{\rm c} H_P$), z is the distance from the convective boundary, H_P is the pressure scale height at the convective boundary, f is a dimensionless free parameter, and $\alpha_{\rm MLT}$ is the mixing length. Relevant to this discussion, CBM increases the mass of the He-core at the end of the main sequence, increases the main sequence lifetime and the luminosity of the core He-burning phase, for a given initial mass and chemical composition.

The inclusion of CBM during the core H-burning phase solves the so-called 'Cepheid mass discrepancy' (see, e.g. Cassisi & Salaris, 2011, for a brief discussion). By comparing results of theoretical pulsation calculations with the period, the estimated mean absolute magnitude and effective temperature of a Cepheid variable, a 'pulsational mass' can be derived. At the same time, a comparison of stellar evolution models with just the estimated mean absolute magnitude and effective temperature, a so-called evolutionary mass, can be assigned to the same object. Traditionally these two independent mass estimates did not agree, the pulsational masses being smaller by 20%–40% if CBM is not included in the models.

Observations of a classical Cepheid in the well-detached, double-lined, eclipsing binary OGLE-LMC-CEP0227 (Pietrzyński et al., 2010) have recently provided the first determination of the dynamical mass (around $4.0\,{\rm M}_\odot$) of a Cepheid to an unprecedented 1% accuracy. This dynamical mass – that is consistent with the pulsational mass – also agrees with the evolutionary mass when main sequence CBM is included in the model calculations, with $\lambda = 0.2$ (Cassisi & Salaris, 2011).

A similar agreement can also be achieved when including the effect of rotation and rotational mixing with inefficient CBM. The element transport associated to rotation (whose treatment contains a number of free parameters and is linked to the treatment of angular momentum transport, also involving free parameters to be calibrated) tends to increase the size of He-cores at the end of the main sequence, the main sequence lifetime, and also the core He-burning luminosity, mimicking the effect of CBM.

Whether CBM or rotational mixing is the main culprit to solve the Cepheid mass discrepancy is difficult to establish given that the efficiencies of both these mechanisms are regulated by free parameters that need to be calibrated. In principle, rotational mixing should also affect the surface abundances of, e.g., nitrogen, that should be enhanced in Cepheid stars if rotation plays a major role. However, as already said, predictions are very uncertain. For example, when comparing models from Ekström et al. (2012) with models from Choi et al. (2016) for a typical Cepheid mass of $9\,M_{\odot}$ and solar initial chemical composition, with (the same initial rotation rates) and without rotation, the effect of rotation on the Hertzsprung-Russell diagram and surface chemical abundances is much larger in Ekström et al. (2012) calculations.

Some breakthrough new constraints on the efficiency of CBM and rotational mixings are starting to come from asteroseismic observations (see, e.g., Moravveji et al., 2015), and will undoubtedly help clarifying the efficiency of these mechanisms, and the role they play in the calculation of Cepheid evolutionary models.

Acknowledgements. I would like to thank the organizers for their kind invitation and the marvelous hospitality during the conference.

References

Andrássy, R., Spruit, H. C., *A&A* **559**, A122 (2013), `arXiv: 1310.8117`

Andrássy, R., Spruit, H. C., *A&A* **578**, A106 (2015), `arXiv: 1502.05628`

Beaton, R. L., et al., *ApJ* **832**, 210 (2016), `arXiv: 1604.01788`

Bossini, D., et al., *MNRAS* **453**, 2290 (2015), `arXiv: 1507.07797`

Bossini, D., et al., *MNRAS* **469**, 4718 (2017), `arXiv: 1705.03077`

Caputo, F., et al., *ApJ* **340**, 241 (1989)

Cassisi, S., Salaris, M., *ApJ* **728**, L43 (2011), `arXiv: 1101.0394`

Cassisi, S., Salaris, M., Irwin, A. W., *ApJ* **588**, 862 (2003), `arXiv: astro-ph/0301378`

Cassisi, S., et al., *A&AS* **134**, 103 (1999), `arXiv: astro-ph/9811329`

Castellani, V., Chieffi, A., Tornambe, A., Pulone, L., *ApJ* **296**, 204 (1985)

Catelan, M., *Ap&SS* **320**, 261 (2009), `arXiv: astro-ph/0507464`

Choi, J., et al., *ApJ* **823**, 102 (2016), `arXiv: 1604.08592`

Constantino, T., Campbell, S. W., Lattanzio, J. C., *MNRAS* **472**, 4900 (2017), `arXiv: 1709.06381`

Constantino, T., Campbell, S. W., Lattanzio, J. C., van Duijneveldt, A., *MNRAS* **456**, 3866 (2016), `arXiv: 1512.04845`

Constantino, T., et al., *MNRAS* **452**, 123 (2015), `arXiv: 1506.01209`

Dalessandro, E., et al., *MNRAS* **410**, 694 (2011), `arXiv: 1008.4478`

Dorman, B., Rood, R. T., *ApJ* **409**, 387 (1993)

Ekström, S., et al., *A&A* **537**, A146 (2012), `arXiv: 1110.5049`

Freytag, B., Ludwig, H.-G., Steffen, M., *A&A* **313**, 497 (1996)

Gabriel, M., Noels, A., Montalbán, J., Miglio, A., *A&A* **569**, A63 (2014), `arXiv:1405.0128`

Herwig, F., *A&A* **360**, 952 (2000), `arXiv:astro-ph/0007139`

Moravveji, E., et al., *A&A* **580**, A27 (2015), `arXiv:1505.06902`

Origlia, L., et al., *A&A* **564**, A136 (2014), `arXiv:1403.4096`

Pietrzyński, G., et al., *Nature* **468**, 542 (2010)

Reimers, D., *Memoires of the Societe Royale des Sciences de Liege* **8**, 369 (1975)

Riess, A. G., et al., *ApJ* **826**, 56 (2016), `arXiv:1604.01424`

Spruit, H. C., *A&A* **582**, L2 (2015), `arXiv:1509.00659`

Straniero, O., Domínguez, I., Imbriani, G., Piersanti, L., *ApJ* **583**, 878 (2003), `arXiv:astro-ph/0210191`

Viallet, M., Meakin, C., Prat, V., Arnett, D., *A&A* **580**, A61 (2015), `arXiv:1506.03100`

BBQ at the Podegrodzie Settlement, at the foot of the Royal Castle in Niepołomice. Gisella Clementini, Marcella Marconi, Matteo Monelli and Maurizio Salaris.