On the maximum mass of differentially rotating compact stars

Magdalena Szkudlarek[1], Dorota Gondek-Rosińska[1],
Loïc Villain[2] and Marcus Ansorg[3]

1. Janusz Gil Institute of Astronomy, University of Zielona Góra, Szafrana 2, 65–516 Zielona Góra, Poland
2. Laboratoire de Mathématiques et Physique Théorique, Univ. F. Rabelais CNRS (UMR 7350), Féd. D. Poisson, 37200 Tours, France
3. Theoretisch-Physikalisches Institut, Friedrich-Schiller-Univerwitat, Jena, Germany

Hot compact stars (neutron stars or strange stars), born in a supernova explosion or being the remnant of neutron star mergers in the binary systems are expected to rotate differentially. Differential rotation may temporally stabilise a massive rotating star against prompt collapse to a black hole. The maximum mass is a crucial parameter being a boundary between stable compact stars and black holes. We summarise our results on the maximum mass of the differentially rotating Strange Quark Stars (SQS) and Neutron Stars (NS) described by a polytropic Equation of State (EOS). Using highly accurate, relativistic multidomain pseudo-spectral code we performed calculations of differentially rotating compact stars for broad ranges of the degree of differential rotation and maximum density. We show that the resulted maximum mass depends on both the degree of differential rotation and the type of solution for all considered EOS. When the differential rotation is taken into account, the maximum allowed mass of NS or SQS can be even 3-4 times higher than the maximum mass of nonrotating configurations. Such result is obtained for moderate or low degree of differential rotation for NS and SQS respectively. We find the universal relation between the maximum mass of a compact star and the degree of differential rotation. These results could have important consequences for gravitational wave astronomy.

1 Introduction

The maximum mass of differentially rotating neutron stars has been already the topic of many studies (e.g. Baumgarte et al., 2000; Lyford et al., 2003; Morrison et al., 2004, and Weih et al. 2018). The topic was recently improved with the relativistic, highly accurate and stable FlatStar code (Ansorg et al., 2009; Gondek-Rosińska et al., 2017; Studzińska et al., 2016) which allowed a deeper understanding of the general structure of the solution space for constant density compact stars and polytropes. The important result obtained with FlatStar was the discovery of four *types* of rotating star's configurations, classified as A, B, C and D, that coexist with each other even for reasonable profiles of angular momentum. In the paper by Gondek-Rosińska et al. (2017) the maximum mass and various other astrophysical parameters were calculated for the first time, for all types of axisymmetric and stationary models of differentially rotating neutron stars described by a $N = 1$ polytropic EOS. For this particular EOS, the maximum mass of differentially rotating neutron star was shown to depend not only on the degree of differential rotation, but also on the type of the configuration. It was shown that the maximum mass is the increasing function of the degree of differential rotation only for type A solution, and decreasing for types B, C and D. It turned out that the highest increase of stellar mass

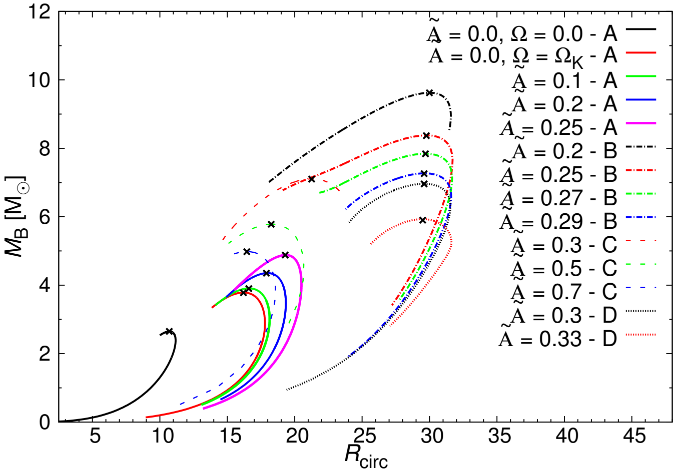

Fig. 1: Baryon mass M_B as a function of circumferential radius R_circ for four types of differentially rotating SQS described by the MIT bag model. Solid black line corresponds to the sequence of non-rotating stars while the red solid line to rigidly rotating stars at the mass-shedding limit. Differentially rotating SQS of type A (for $\tilde{A} = 0.1, 0.2$ and 0.25 are plotted as solid lines, dashed-dotted lines represent type B solution, while dashed lines and dotted lines type C and D respectively. Black crosses are the maximal masses obtained for each sequence.

(4 times the maximum mass of non-rotating neutron star) was obtained for one of the new types of solutions, type B, but for a modest degree of differential rotation. The study of (Studzińska et al., 2016) confirmed that the same picture holds for neutron stars modeled by polytropes with different adiabatic indexes. In this paper, we present results of calculations of axisymmetric and stationary relativistic models of differentially rotating NS described by polytropes, and SQS for broad ranges of the degree of differential rotation.

2 Model calculations and results

We performed calculations of axially symmetric and stationary relativistic models of differentially rotating NS and SQS using a modern code based on double-domain pseudo-spectral method given by Ansorg et al. (2009). To describe SQS we use the MIT bag model EOS: $P = (\rho - \rho_0)c^2/3$, and NS the polytropic EOS $P = \rho^\Gamma$, where P is pressure, ρ is density, c speed of light Γ is a polytropic index and $\rho_0 = 4.3 \times 10^{14}\,\mathrm{g\,cm^{-3}}$. We adopt an astrophysically motivated rotation law introduced by Komatsu et al. (1989) which was later on considered by many authors (Cook et al., 1992; Baumgarte et al., 2000; Lyford et al., 2003). The most important assumptions of the rotational law used in this paper are:

- after Komatsu et al. (1989), our rotational law implies an angular velocity that monotonously decreases from the axis of rotation to the equator;
- the degree of differential rotation is described by a dimensionless parameter $\tilde{A}$ (Ansorg et al. (2009)), the higher $\tilde{A}$ is the higher degree of differential rotation;

- $\tilde{A} = 0$ corresponds to rigid rotation, $\Omega = \Omega_c$ in the whole star;
- for $\tilde{A} = 1$, the angular velocity at the equatorial surface is around half Ω_c.

We found that the solution space of differentially rotating SQS is divided into four types of solutions as in the case of NS (see Ansorg et al., 2009; Studzińska et al., 2016; Gondek-Rosińska et al., 2017)). Depending on the degree of differential rotation $\tilde{A}$, maximum density $\rho_{\rm max}$, and on the third chosen parameter, i.e. polar and equatorial radius ratio r_p/r_e, the resulted configuration can be of type A, B, C or D. There is a specific critical value of degree of differential rotation $\tilde{A}_{\rm crit}$ characteristic for a given maximum density, which divides solution space into these four different types:

- $0 < \tilde{A} < \tilde{A}_B$ - all sequences are of type A;
- $\tilde{A}_B < \tilde{A} < \tilde{A}_{\rm crit}$ - all sequences are of type A or B;
- $\tilde{A}_{\rm crit} < \tilde{A} < \tilde{A}_D$ - all sequences are of type C or D;
- $\tilde{A}_D < \tilde{A}$ - all sequences are of type C.

Types B and D of the solution exist only due to differential rotation, for small values of $r_p/r_e < 0.3$ and do not have direct connection with static configuration ($\tilde{A} = 0$, $r_p/r_e = 1$).

The maximum mass of a relativistic star helps determining if an observed compact object is a black hole or a material celestial body. It is a key-factor for establishing the life span of the short-lived remnant born from the merger of a compact binary system (Baumgarte et al., 2000; Shibata & Uryū, 2002; Shibata et al., 2005; Giacomazzo et al., 2011).

The baryon mass dependence on circumferential radius for four types of solutions of SQS is shown in Fig. 1. Each line corresponds to a sequence of stars with fixed degree of differential rotation and each type of line corresponds to a different type of the four solutions labeled at the figure. For each sequence the configuration the maximum computed mass is shown as a black cross. The maximum mass of differentially rotating SQS, similarly to NS is reached at different points in the solution space depending on the solution type: A, B, C, D.

- For type A maximum mass is reached near the mass-shedding limit but not exactly at it;
- For type B maximum mass is at mass-shedding limit;
- For type C maximum mass is reached for toroidal configurations ($r_p/r_e \to 0$);
- Type D starts and ends at mass-shedding limit but the maximum mass configuration lies at the end with smaller value r_p/r_e.

We found that the maximum allowed mass of differentially rotating stars depends on the degree of differential rotation and on the type of the solution. It is an increasing function of the degree of differential rotation for type A solution but decreasing for other types B, C and D. This is an universal relations for all EOS considered by us. The highest increase of maximum mass of differentially rotating SQS is reached for type B solution, represented by dotted-dashed lines at Fig. 1, and for the small degree of differential rotation, $\tilde{A}$ 0.2, smaller than in the case of NS described by the polytropic EOS.

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