Nicolaus Copernicus' gnomonic array for Sun observation

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This article presents studies of residues of the specific sundial painted in the wall of the cloister in the northeast part of the castle in Olsztyn. The table was made about 1517, when Nicolaus Copernicus held the office of the administrator of Warmian Chapter properties located around the town. The results following from the mathematical model of that observational instrument are discussed and a novel insight into its functioning and accuracy is provided.

1 The astronomical table

The construction of the astronomical array is characterized by substantial originality. Due to its location, on the north-eastern wall of the cloister of the castle (Fig. 1), its author could not copy the well-known sundial patterns, but was forced to use a novel method employing reflection of the rays of the Sun. It is likely that, as noted by Przypkowski (1973, 1959), this application of the Sun's reflection was the first case in the history of gnomonics. The astronomical table measures 7.05 m by 1.4 m, and it was imprinted onto a wet fresco whitewash, used probably around 1517 (Dzieciątkowska, 2013).

2 The calendar lines

In order to define the astronomical, geographical and architectural determinants of the function of the gnomon array, it is convenient to adopt a horizontal coordinate system. The mirror, as the essential element of this instrument, is assumed to be in the centre of the celestial sphere in the plane of the horizon. The axis of the celestial sphere is inclined according to the latitude of the place of observation $\varphi = 53°46'36''$. The local meridian passes through the celestial poles, the zenith, and the nadir and defines the north and south points in the plane of the horizon. The ecliptic is inclined towards the plane of the celestial equator at an angle of $23.5°$, intersecting the equator at two points, which correspond respectively to the vernal equinox (the point of Aries) and the autumnal equinox (the point of Libra). The Sun, as viewed from the Earth, apparently moves along the ecliptic eastward at an average speed of $0.986°\,\mathrm{d}^{-1}$, In the plane of horizon the perpendicular line from the mirror to the vertical plane of the table defines the origin of the Cartesian coordinate system (x, y, z). The instantaneous position of the Sun determined its azimuth A and its high h over horizon. The reflected sunray light is entering the plane of the table at the point (x, z). However, the coordinates (A, h) of the Sun change rapidly in the horizontal frame, and moreover both are the complex functions of time. Therefore, to analyze the functioning of the instrument, it is more convenient to use the equatorial

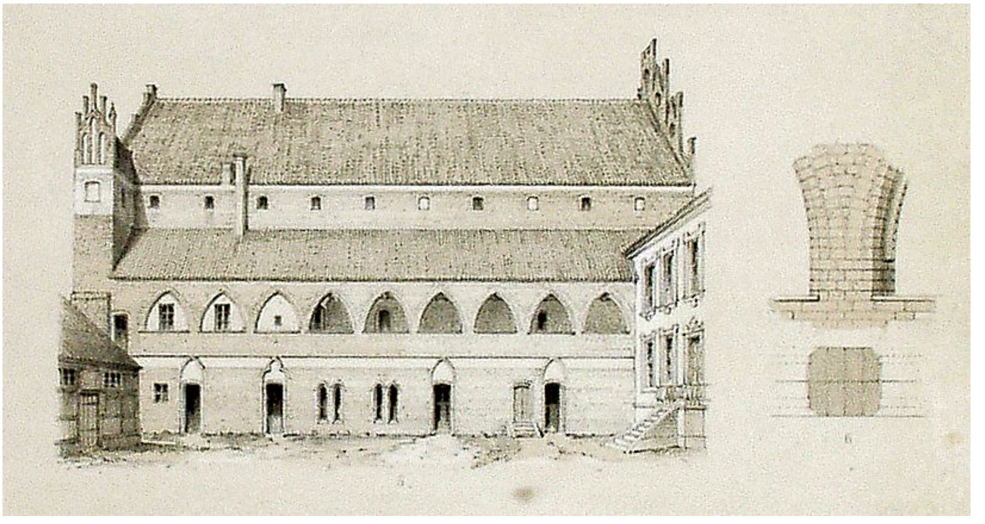

Fig. 1: The view of the northeast side of the castle courtyard in 1852. The residues of the astronomical table are in the second-floor cloister behind bricked-up arches. The sketch by Ferdinand von Quast, the first Prussian conservator of historical monuments. Courtesy of the Museum of Warmia and Mazury in Olsztyn.

coordinate system with coordinates of declination δ and hour-angle t. The pair (δ, t) applied to the position of the Sun reflects both the direction of the rotation axis of the Earth in space and its current state of daily rotational motion and facilitates the description of the calendar lines in the table (Meeus, 1991).

During the day the declination of the Sun changes slightly compared to the change of the hour angle. Eliminating the hour angle from the equations we get a mapping of the daily path of the Sun on the skies, which depends only on its declination. In the particular case of the orientation of the table, which is perpendicular to the local meridian ($\theta = 0°$), the daily pathway of the reflected light onto the (x, z) plane is hyperbolic (Fig. 2). The graphical manifold of these curves is similar to double-edged axe, whose Greek name is pelekinon. However, the actual azimuth of the wall is $A_{\mathrm{T}} = 123°33'36''$ (Miałdun, 2009). This results in distortion of hyperbolae stretching their shapes along the wall of the cloister.

The mathematical functions that account for this distortion were fitted to the residues of the original lines plotted on the plaster. For that purpose, the coordinates of some points on each day line were digitized (Tro, 2015), using a high-resolution photogrammetric image of the table (Miałdun, 2006). The fitting of the non-linear function to the data set was accomplished using a least-squares Levenberg-Marquardt algorithm (Ph. R. Bevington, 2003). The calculations were implemented in the Maple 12 (Map, 2009), using a freely available software procedure (D. E. Holmgren & Monagan, 2004). The standard deviation of the fit to each line was adopted as the criterion of the quality of the fit. The results are shown in Tab. 1. The line No. 9 corresponding to the equinox is marked. The slope of the equinox line measured on the table by matching an arbitrary linear function is: $\alpha = 21°14'7'' \pm 4''$.

The position of the mirror below the lower edge of the table that results from the fitting procedure is $z_0 = -1.864$. The range of the ecliptic longitudes given

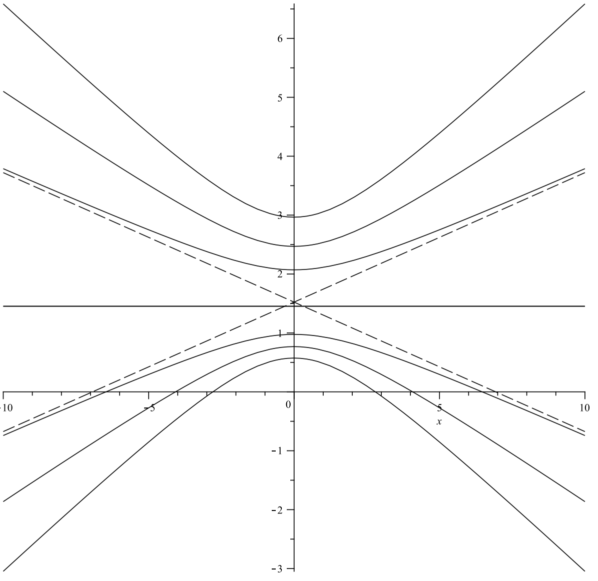

Fig. 2: Hyperbolic paths of the light spot for orientation of the table perpendicular to the meridian ($\theta = 0^\circ$) and $-20^\circ \leq \delta \leq 20^\circ$. The horizontal straight line corresponds to the path on the equinox when ($\delta = 0^\circ$). The curves below the horizontal line correspond to observations made before the vernal equinox, above this line, those made after it.

No. of line	Declination $\delta[^\circ]$	Error $\pm\Delta\delta[^\circ]$	MSE	Ecliptic $long.\lambda[^\circ]$	Error $\pm\Delta\lambda[^\circ]$
1	-15.490	0.077	0.031	317.824	0.250
2	-13.758	0.074	0.039	323.282	0.224
3	-11.955	0.042	0.025	328.617	0.120
4	-10.209	0.042	0.024	333.540	0.116
5	-8.690	0.023	0.014	337.677	0.062
6	-6.651	0.030	0.030	343.072	0.078
7	-4.395	0.035	0.035	348.892	0.089
8	-2.430	0.021	0.021	353.881	0.053
9	-0.001	0.020	0.008	359.998	0.050
10	1.834	0.029	0.012	4.615	0.073
11	3.705	0.023	0.011	9.349	0.058
12	5.766	0.021	0.009	14.630	0.054
13	7.467	0.014	0.006	19.069	0.037
14	9.076	0.012	0.004	23.364	0.032

Table 1: The values of declination δ of the Sun and its corresponding ecliptic longitudes λ for consecutive calendar lines numbered from the left side of the table (see Fig. 3).

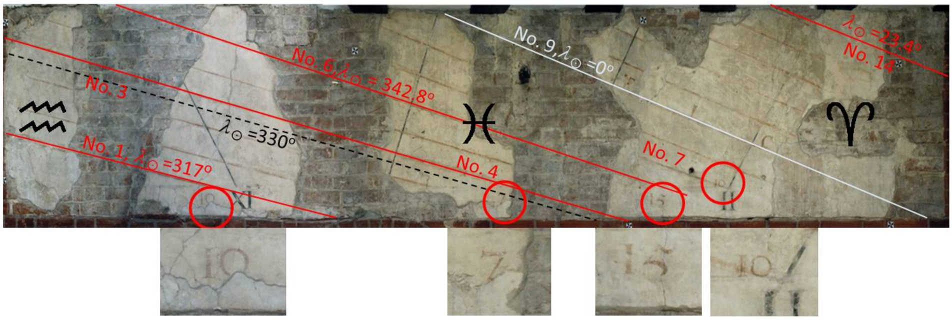

Fig. 3: View of the astronomical table. The day line numbering convention used is from left to right. The red circles indicate the markings of the lines that are relevant for explanation of the operation of the array. Enlarged fragments of these inscriptions are shown below the table. Photographs courtesy of the Museum of Warmia and Mazury in Olsztyn.

in Tab. 1 indicates that the range of λ covered by the board corresponds to two and nearly a quarter signs of the zodiac (Fig. 3). The dates corresponding to end portion of the sign of Aquarius for $\lambda_\odot < 330°$ appears in the left part of the table, while in the right part there is a fragment of calendar that correspond to the sign of Aries, ranging from $\lambda_\odot = 0°$ to $\lambda_\odot = 23.4°$. The central section of the array includes the sign of Pisces, in $330° < \lambda_\odot < 360°$. The average distance in ecliptic longitude $\Delta\lambda$ between two adjacent calendar lines is $5.00° \pm 0.16°$. On this basis, one can identify the meaning of the Arabic numerals that describe the day lines at the bottom margin of the astronomical table (Dzieciątkowska, 2013). Line No. 9 corresponding to the vernal equinox occupies a central position on the board (see Fig. 3). The remnants of blue colour and the inscription "T I C", which appears just above this line distinguish it from the other red lines. The simplest explanation would of the significance of this inscription seems to be that it is an abbreviation of the label designating the beginning of the reckoning time of the solar year: Tropical Initium Calendarium.

3 The estimation of the accuracy

To estimate the accuracy of the observational determination of the equinox moment, the individual calendar lines are presented as points (T_i, λ_i) on a plane with the coordinates: time in Julian Days and ecliptic longitude of the Sun. The points forming pairs situated on two branches of the function of the apparent geocentric longitude of the Sun are selected and connected by straight lines. The intersection of these lines defines the observationally determined instant of the equinox (Fig. 4).

The cross point in Fig. 4 indicates the mean value of the moment of equinox determined observationally and error bars indicate corresponding standard deviations. The covariance ellipses for one σ, is represented by solid line, while the 95% confidence ellipse by dotted line. The vertical dash-dot line indicates the theoretically determined instant of the equinox of 1517 in Olsztyn's LMT, while the dashed one corresponds to the same moment in UTC.

It is interesting to compare the mean value of the time of the equinox resulting from the studies of day lines recorded on the astronomical table with the value for the

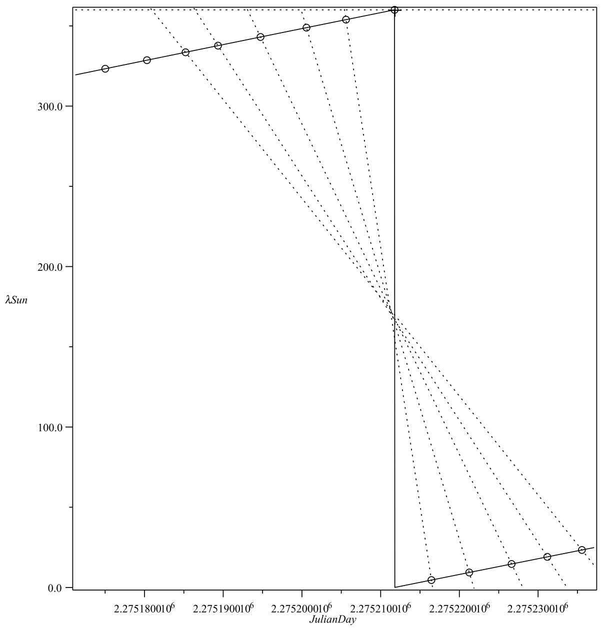

Fig. 4: The dash-dot line represents the function of the ecliptic longitude of the Sun, near the vernal equinox of 1517. The boxes denote the observations (T_i, λ_i) made before and after the equinox.

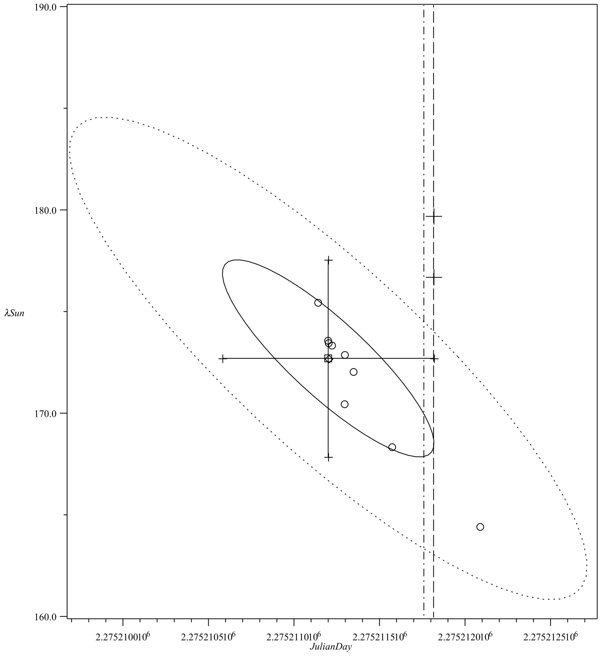

Fig. 5: Magnified view of the area of intersection of lines (the circles).

year 1517 computed on the basis of the Meeus algorithm (Meeus, 1991). According to such calculations, the spring equinox in 1517, occurred on Sunday, March 11, in Julian calendar, at $6^{\mathrm{h}}8^{\mathrm{m}}58.579^{\mathrm{s}}$ UTC (Smith, 2016). Taking into account the offset $\Delta T = 1^{\mathrm{h}}21^{\mathrm{m}}53.832^{\mathrm{s}}$ from UTC, which results from the location of the astronomical array at geographical longitude $\lambda = 20.4743^{\circ}$ E, the equinox should take place the day before, at $16^{\mathrm{h}}43^{\mathrm{m}}38.44^{\mathrm{s}}$ LMT, so that it precedes the theoretically determined instant by $14^{\mathrm{h}}47^{\mathrm{m}}13.96^{\mathrm{s}}$. As can be seen in Fig. 5, all but one of the points of intersections are grouped to one side of the vertical dash-dot line indicating the moment of equinox. This demonstrates systematic errors, which result, probably, from the regular prior overestimation of the observed ecliptic longitude of the Sun when compared with the corresponding calculated value.

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