Mode Identification in Rapidly Rotating Stars from BRITE Data

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Apart from recent progress in Gamma Dor stars, identifying modes in rapidly rotating stars is a formidable challenge due to the lack of simple, easily identifiable frequency patterns. As a result, it is necessary to look to observational methods for identifying modes. Two popular techniques are spectroscopic mode identification based on line profile variations (LPVs) and photometric mode identification based on amplitude ratios and phase differences between multiple photometric bands. In this respect, the BRITE constellation is particularly interesting as it provides space-based multi-colour photometry. The present contribution describes the latest developments in obtaining theoretical predictions for amplitude ratios and phase differences for pulsation modes in rapidly rotating stars. These developments are based on full 2D non-adiabatic pulsation calculations, using models from the ESTER code, the only code to treat in a self-consistent way the thermal equilibrium of rapidly rotating stars. These predictions are then specifically applied to the BRITE photometric bands to explore the prospects of identifying modes based on BRITE observations.

1 Introduction

Rapidly rotating stars intervene in many areas of astrophysics. For instance, the majority of massive and intermediate mass stars are rapid rotators (*e.g.* Zinnecker & Yorke, 2007; Royer, 2009). Primordial stars are also expected to be rapid rotators due to their low metallicity and hence, opacity (*e.g.* Ekström et al., 2008). Rapid rotation is also thought to play a key role in the precursors to gamma-ray bursts (*e.g.* Woosley & Heger, 2006). Understanding these stars would yield valuable information to these areas of astrophysics.

Rapid rotation introduces many new phenomena in stars. These include centrifugal deformation, gravity darkening, baroclinic flows, and various forms of turbulence and transport phenomena (*e.g.* Maeder, 2009; Rieutord et al., 2016). As a result, there are many uncertainties in the models and a need for further observational constraints. Currently, one of the best ways of constraining stellar structure is through asteroseismology. However, identifying pulsation modes in rapidly rotating stars, *i.e.* finding the correspondence between observed and theoretical pulsations, is a formidable challenge (*e.g.* Goupil et al., 2005). Indeed, the pulsation spectra lack simple frequency patterns, and reliable predictions for mode amplitudes are not available given that modes in these stars tend to be excited by the κ mechanism. As a result, it is necessary to apply mode identification techniques.

There are two main types of mode identification techniques. The first is based on multi-colour photometry and involves looking at ratios between pulsation amplitudes, and phase differences, in different photometric bands. These signatures depend on the geometry of the pulsation mode but are independent of the intrinsic mode amplitudes. With its two colours, the BRITE mission is an ideal source of space-based multi-colour photometric observations of pulsating stars. The second approach is based on spectroscopy and consists in looking at how the profile of a given absorption line changes with time. These variations are known as line profile variations (LPVs) and provide a very rich information which can be complementary to that provided by the photometric approach. Both of these techniques need to be adapted to rapid rotation.

In the following, we will focus on photometric mode identification techniques. We will specifically look at results in the BRITE photometric bands and see up to what extent mode identification may be constrained. The next section describes the prerequisites for coming up with reliable predictions as well as the calculations carried out. This is followed by various results which focus on amplitude ratios, phase differences, and complex asteroseismology. A discussion concludes these proceedings.

2 Calculating mode visibilities

In order to calculate reliable mode visibilities, it is necessary to carry out 2D pulsation calculations which fully take into account the effects of rapid rotation, in order to correctly calculate the geometry of the modes. For instance, low-degree acoustic modes become island modes at rapid rotation rates (Lignières & Georgeot, 2008, 2009). These modes take on an elongated structure which circumvents the equator and is characterised by a new set of quantum numbers ($\tilde{n}$, $\tilde{\ell}$, m) (*e.g.* Reese, 2008). Calculating such modes as well as other modes present in rapidly rotating stars requires the use of stellar models which fully take into account stellar deformation. Furthermore, non-adiabatic pulsation calculations are required in order to correctly calculate $\delta T_{\rm eff}/T_{\rm eff}$, the variations in effective temperature, as these intervene in the intensity variations used to calculate mode visibilities. In order to carry out such calculations, it is necessary for the stellar model to respect the energy conservation equation. This means that the stellar model will be baroclinic, *i.e.* isobars, isotherms, and isochores will not coincide, and the rotation profile will be non-conservative, *i.e.* it will depend on both s, the distance to the rotation axis, and on z, the vertical coordinate.

In the work presented here, we will use ESTER[1] models (Espinosa Lara & Rieutord, 2013; Rieutord et al., 2016) as these are currently the only rapidly rotating models which satisfy the energy equation locally. Non-adiabatic calculations will be carried out using the TOP[2] pulsation code (*e.g.* Reese et al., 2009, 2017a). For stars in the mass range of δ Scuti stars, our implementation of non-adiabatic calcu-

[1]Evolution STEllaire en Rotation.
[2]Two-dimensional Oscillation Program.

lations are not fully reliable. Accordingly, we will also use models from the SCF[3] code (Jackson et al., 2005; MacGregor et al., 2007) along with adiabatic pulsation calculations. Non-adiabatic effects will be approximated in the same way as is done in Reese et al. (2017b), *i.e.* using the pseudo non-adiabatic (PNA) approach (see Table 1).

3 Multi-colour photometric mode identification

3.1 Amplitude ratios

As was shown in Daszyńska-Daszkiewicz et al. (2002) and Townsend (2003) amplitude ratios depend on the azimuthal order, m, and the inclination, i, in rotating stars, thereby complicating the task of mode identification. Nonetheless, Reese et al. (2013) found similar amplitude ratios for modes with the same (ℓ, m) values but different radial orders, n, as expected from ray theory (Pasek et al., 2012). Accordingly, Reese et al. (2017b) proposed an alternate mode identification strategy. This strategy involves choosing a reference mode, then choosing the N (typically 9) other modes with the most similar amplitude ratios. When the reference mode happens to be an island mode, the other modes also tend to be island modes with similar quantum numbers. The corresponding frequencies would then follow patterns as expected from the asymptotic frequency formula (*e.g.* Lignières & Georgeot, 2009; Reese et al., 2009). By repeating this procedure, one can hope to group similar modes together into families and identify recurrent frequency spacings as expected from the asymptotic formula. An open question is whether this strategy still continues to work when using the 2 photometric bands from BRITE rather than the 7 bands from the Geneva photometric system.

In Table 1, we give the average success rates at finding other island modes using the Geneva and BRITE photometric systems for different stellar masses. The third column gives the success rate at identifying other island modes if the reference mode is an island mode, whereas the fourth and fifth columns give the success rates at finding island modes with the same $(\ell, |m|)$ and $(\tilde{\ell}, |m|)$ values, respectively. We recall that two modes with the same $(\tilde{\ell}, |m|)$ values will not necessarily have the same $(\ell, |m|)$ values as one could be symmetric with respect to the equator and the other anti-symmetric. The last column gives the proportion of island modes in the entire set of modes. As can be seen, the success rates for the BRITE photometric system are much lower. Typically, one can expect to identify 1 or 2 other island modes out of a set of $n = 9$ modes, which is insufficient for the purposes of mode identification.

One may then wonder what happens if a supplementary photometric band is included. For reasons of normalisation, we prefer not to mix visibilities from the Geneva and BRITE systems. Hence, we use the B and G Geneva bands as representative of the BRITE bands although we do note the latter are much wider. Then we include either the U or V1 bands as these are centred around the smallest and highest wavelengths, respectively, besides the B and G bands. Table 2 gives the success rates for the BRITE photometric system as well as reduced versions of the Geneva system. Columns 2 to 4 have the same meaning as columns 3 to 5 of Table 1. As can be seen, adding one band, especially at small wavelengths, increases the success

[3]Self-Consistent Field.

Table 1: Success rates for the mode identification strategy using the Geneva and BRITE photometric systems, for SCF models at $0.6\,\Omega_{\rm K}$ (where $\Omega_{\rm K}$ is the Keplerian break-up rotation rate).

Model	**Photo. System**	..... **Success rates** ..... **Island**	$(\ell,\,|m|)$	$(\tilde{\ell},\,|m|)$	**Island prop.**
Adia. ($2\,{\rm M}_\odot$)	Geneva	0.564	0.359	0.416	0.0115
Adia. ($2\,{\rm M}_\odot$)	BRITE	0.145	0.058	0.071	0.0115
Adia. ($1.8\,{\rm M}_\odot$)	Geneva	0.554	0.401	0.452	0.0330
Adia. ($1.8\,{\rm M}_\odot$)	BRITE	0.258	0.133	0.159	0.0330
PNA ($1.8\,{\rm M}_\odot$)	Geneva	0.469	0.303	0.349	0.0330
PNA ($1.8\,{\rm M}_\odot$)	BRITE	0.201	0.079	0.102	0.0330

PNA = pseudo non-adiabatic

Table 2: Success rates for the mode identification strategy for the BRITE and reduced versions of the Geneva photometric systems. These values are obtained for the $1.8\,{\rm M}_\odot$ stellar model using pseudo non-adiabatic calculations.

Photo. Bands	..... **Success rates** ..... **Island**	$(\ell,\,|m|)$	$(\tilde{\ell},\,|m|)$
BRITE	0.201	0.079	0.102
B, G	0.182	0.060	0.087
U, B, G	0.327	0.182	0.211
B, V1, G	0.285	0.146	0.187

rates appreciably.

In summary, this approach is expected to start working for at least 3 photometric bands, and would also require a large number of acoustic modes, preferably in the asymptotic regime. Hence, additional observations besides those of BRITE are needed, and δ Scuti stars would be the most suitable targets.

3.2 Amplitude ratios and phase differences

In some cases, nonetheless, observations may only be available in 2 rather than 3 bands. Furthermore, the number of observed modes may be too small for the above strategy. This raises the question as to how much information can be obtained from both amplitude ratios and phase differences. In what follows, we use full non-adiabatic calculations as these are needed for obtaining reliable phase differences. Accordingly, we will work with $9\,{\rm M}_\odot$ ZAMS models produced by the ESTER code, with $X = 0.700$, $Z = 0.025$, and rotation rates ranging from 0.0 to $0.5\,\Omega_{\rm K}$ (where $\Omega_{\rm K} = \sqrt{GM/R_{\rm eq}^3}$ is the Keplerian break-up rotation rate, $R_{\rm eq}$ being the equatorial radius).

As a first step, we compared our amplitude ratios vs. phase differences with those from Handler et al. (2017, hereafter H17) for similar non-rotating models in order

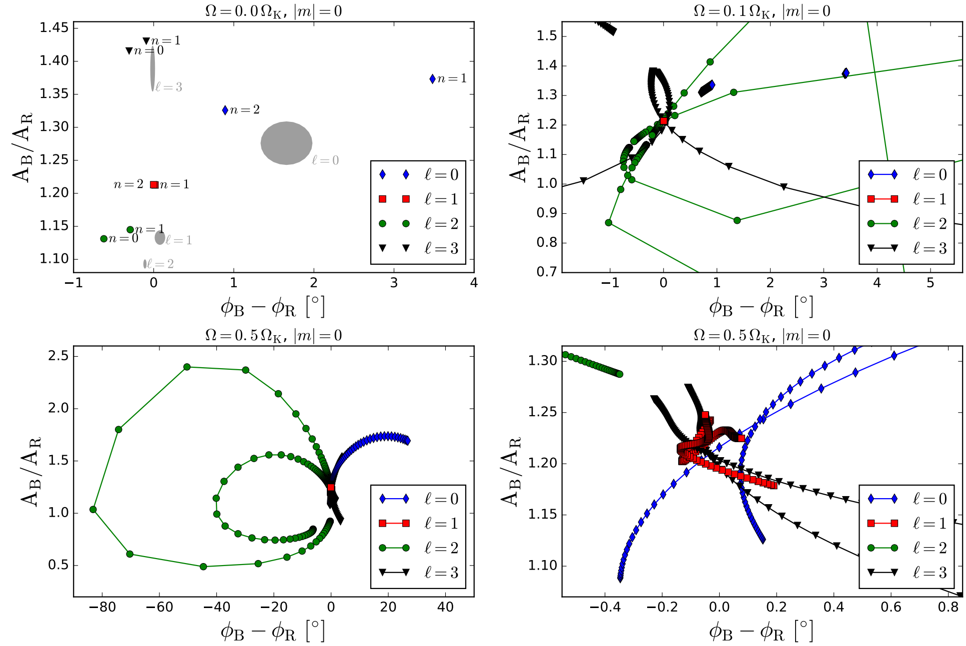

Fig. 1: Amplitude ratios vs. phase differences for axisymmetric modes in $9\,M_\odot$ ESTER models at different rotation rates using the BRITE photometric bands. The lower right panel is a zoom in of the lower left panel. Lines connect results for the same $(n,\,\ell)$ values but for stellar inclinations ranging from 2° to 89° in increments of 1°.

to validate our calculations. The mass of our model is $9\,M_\odot$ whereas those of H17 range from 9.5 to $10\,M_\odot$. The amplitude ratios vs. phase differences are plotted in the upper left panel of Fig. 1, the grey regions corresponding to H17. As can be seen, a qualitative agreement is obtained.

We then look at how rotation affects amplitude ratios and phases differences in the remaining panels of Fig. 1. As expected, these now depend on the stellar inclination, in contrast to the non-rotating case. Furthermore, there are large excursions in these diagrams. This is typically caused by mode amplitudes going to zero at slightly different stellar inclinations in the different photometric bands. Accordingly, it seems unlikely that such excursions will be seen in observed stars, since at least one of the components will likely be below the detection threshold.

An important question is then whether amplitude ratios and phase differences will be similar for modes with the same $(\ell,\,|m|)$ values but different radial orders, and whether this can help with mode identification. Figure 2 shows amplitude ratios and phase differences for $(\ell,\,|m|) = (3,2)$ in the left panel and $(2,0)$ in the right panel. As can be seen, the left panel corresponds to a case where the amplitude ratios and phases differences are similar, whereas the right panel shows a case with larger differences, especially for $i \simeq 30°$, due to large excursions at different inclinations. Hence, the answer to the question depends both on the choice of $(\ell,\,|m|)$ and on the inclination. A more exhaustive search for a large set of radial orders will be needed

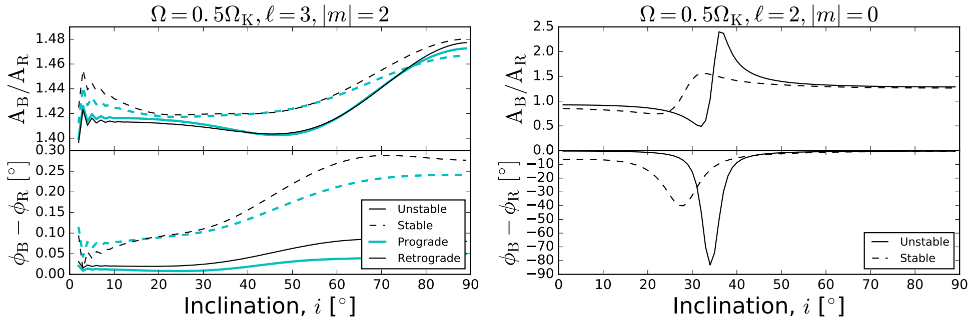

Fig. 2: Amplitude ratios and phase differences as a function of inclination for modes with the same (ℓ, $|m|$) values. We note that the zigzags at low inclinations in the left panel are numerical artifacts due to low mode visibilities in both bands.

for δ Scuti type stars once reliable non-adiabatic pulsation calculations are available for these.

Even with a limited number of modes, one can always apply a χ^2 minimisation to find the modes which best reproduce observed amplitude ratios and phases differences. In this regard, we recall the work done by Daszyńska-Daszkiewicz et al. (2015) in modelling the SPB star μ Eridani, where such a minimisation was carried out in the framework of the traditional approximation, only including excited modes, and taking into account the observed $v \sin i$ value. This allowed them to constrain the mode identification as well as the values of v and i.

3.3 Complex asteroseismology

In 2009, Daszyńska-Daszkiewicz & Walczak applied what they called "complex asteroseismology" to the β Cephei star θ Ophiuchi. Complex asteroseismology involves observationally determining the f parameter in addition to the mode identification, using both multi-colour photometry and radial velocity measurements from spectroscopy, where f is the ratio of the bolometric flux perturbations to the radial displacement:

$$\frac{\delta\mathcal{F}_{\rm bol}}{\mathcal{F}_{\rm bol}} = 4\frac{\delta T_{\rm eff}}{T_{\rm eff}} = f\frac{\xi_r}{R}. \tag{1}$$

The f parameter is complex due to non-adiabatic effects which introduce a phase shift between the flux perturbations and radial displacements. This parameter is independent of latitude in the non-rotating case because $\delta\mathcal{F}_{\rm bol}$ and ξ_r are proportional to the same spherical harmonic. An open question is what happens to f when the star rotates rapidly.

In Fig. 3, we plot the real and imaginary parts of f as a function of colatitude for two different modes. In our definition of f, we used the displacement perpendicular to the stellar surface, $\xi_{\rm v}$, normalised by the equatorial radius. As can be seen f now depends on the colatitude. Furthermore, sharp spikes occur in the right panel as a result of $\delta T_{\rm eff}$ and $\xi_{\rm v}$ going to zero at slightly different colatitudes. Hence, the f parameter should be described as an f profile. This raises the question as to whether it would be possible to define some sort of disk-integrated, possibly

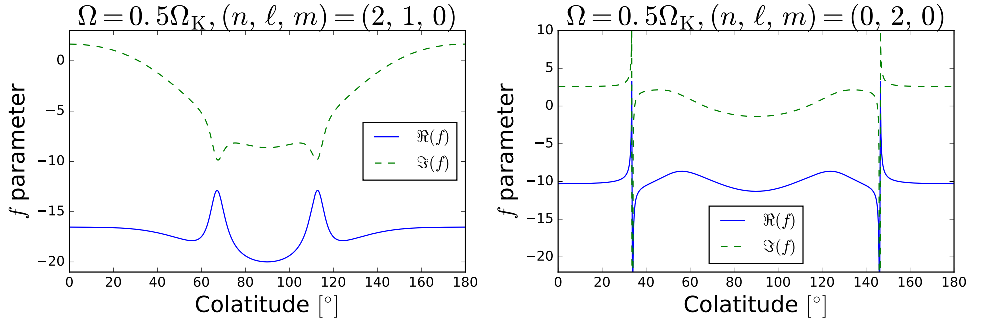

Fig. 3: The f parameter as a function of colatitude for two pulsation modes in a rapidly rotating ESTER model.

inclination-dependent, effective f parameter which may still be used to constrain stellar structure, as is done in the non-rotating case.

4 Conclusion

In summary, rotation leads to a much more complicated picture for amplitude ratio and phase differences. It also leads to a latitude-dependant f parameter, f being the ratio between the bolometric flux perturbations and radial displacement. Accordingly, this makes mode identification more difficult and complicates complex asteroseismology. Conversely, this may provide tighter constraints on the azimuthal orders and stellar inclination (*e.g.* Daszyńska-Daszkiewicz et al., 2015).

Different prospects include extending full non-adiabatic calculations to δ Scuti stars, developing a database of mode visibilities and LPVs for the purposes of identification, adapting and/or developing mode identification tools, and fully exploiting pulsation data of rapidly rotating stars from the BRITE mission as well as from PLATO 2.0 and the ground-based spectroscopic SONG network.

Acknowledgements. DRR and MR acknowledge the support of the French Agence Nationale de la Recherche (ANR) to the ESRR project under grant ANR-16-CE31-0007. DRR also acknowledges financial support from the "Programme National de Physique Stellaire" (PNPS) of CNRS/INSU, France.

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Enjoying Montreal, just before the conference. Milena Ratajczak, Tahina Ramiaramanantsoa, Radek Smolec, Monika Rybicka, Grzegorz Marciniszyn, Bert Pablo and Grzegorz Woźniak