Lecture IV. Dark Matter & particles (WIMP)

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This article summarizes the fourth of a series of lectures delivered at the Cosmology School "Introduction to Cosmology". It reviews different candidates for Dark Matter particles in their historical context.

Do we detect all the baryons?

The thermal history of the Universe, in particular Nucleosynthesis, tell us how many baryons we formed in the early Universe. This means that any mass we detect in excess of what we expect from the baryon genesis must be non-baryonic. The primordial abundances of Deuterium, ^{3}He, and ^{7}Li are strongly dependent on the competition among the various nuclear reaction rates that depends on the density of nucleons and the expansion rate of the Universe. The nucleon density changes with the expansion so that it is convenient to use as a reference the ratio $\eta \equiv \frac{n_{\rm N}}{n_\gamma}$[1] and since, as we have seen this is a rather small number, we define $\eta_{10} \equiv 10^{10}\left(\frac{n_{\rm N}}{n_\gamma}\right) = 274\Omega_{\rm b}h^2$. The value estimated using SBBN (Standard Bing Bang Nucleosynthesis) $\eta_{10} = 6.10^{+0.67}_{-0.52}$, Steigman (2003) is in excellent agreement with other estimates, mainly CBR, $\eta_{10} = 6.14 \pm 0.25$, Spergel et al. (2003). This corresponds respectively to $\Omega_{\rm b}h^2 = \frac{6.14}{274} = 0.0222$ and $\Omega_{\rm b}h^2 = 0.0224$. Tytler et al. (1996) from measurements of D/H derives $\Omega_{\rm b}h^2 = 0.024^{0.006}_{0.005}$. A perused and very representative plot the latter estimate is given by Schramm & Turner (1996), see Fig. 1.

That is with $h = 0.7$ we expect $\Omega_{\rm b} = 0.045$[2] This is a very robust estimate. The evidence nowadays is we do not detect all the baryons we are expecting. The complete inventory of the baryons in the Universe is obviously very important because either there are physical processes that deplete the baryons in part of the Universe or something is wrong with our estimates. Likely, however, the fact is that baryons may exist in some states that are very hard to detect so that our search must continue. For now it also seems that the problem depends also on the scale we are searching. Shull et al. (2012) in their census of a multiphase intergalactic medium at low redshift, $z < 0.7$, estimate that about 30% of the baryons are still undetected in the nearby Universe. Part of the undetected baryons may need new surveys of higher sensitivity and accuracy to measure lines at lower column density. These both for the UV and the X-ray. In the latter passband the detection claimed by Nicastro et al. (2005) has been never duplicated or confirmed by different (by other scientists) analysis of the same data. Also in the X-ray we may need more sensitive instruments, indeed highly ionized atoms need to be detected in this band.

[1]This ratio is accurately preserved after the annihilation e^+e^- so that the value at the Bing Bang Nucleosynthesis should be equal to its value today.

[2]Using the Planck Cosmological Parameters (Planck Collaboration et al., 2016) we have: $\Omega_{\rm b}h^2 =$

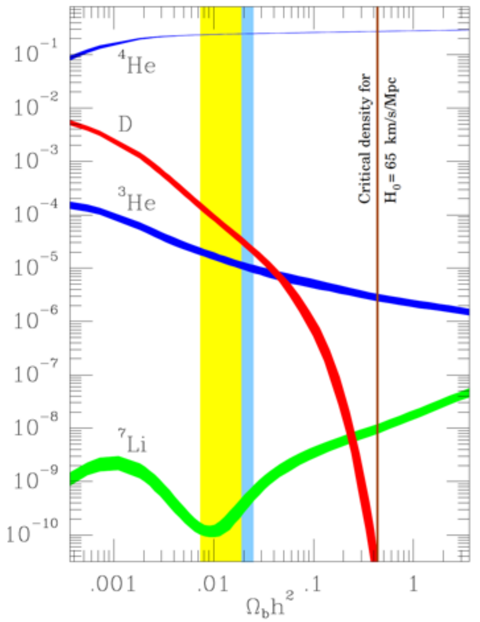

Fig. 1: Big bang production of light elements.

Shull et al. (2012) performed a census of the baryons, Fig. 2. More than a quarter of the baryons detected are the results of the absorption observed in high redshift quasars, the so called Lyman alpha forest and is a direct evidence of the existence and properties of the Intergalactic medium. The UV intergalactic radiation field is strong enough to keep most of the matter (baryonic of course) highly ionized and we have a clumps or regions producing absorption lines. The Lyman absorbers account for 28% of the baryonic matter. A very good tracer of the highly ionized medium is OVI. It seems there are two populations with different densities and temperatures: one at $T \sim 10^{5.5}$ K that is collisional ionized and consists in 55% of the total OVI and one at $T \sim 10^{4.5}$ K that is photoionized 37% of the total while the remainder is in denser gas near galaxies, Smith & Marian (2011). The value given by Shull et al. (2012) is based on the work by Lehner et al. (2007) who find an absorption line frequency per unit redshift $\frac{dN}{dz} = 30 \pm 4$ leading to a 20% of baryons and on the work by Danforth et al. (2010) who found a frequency $\frac{dN}{dz} = 18 \pm 11$.

For the smaller contributions by other cosmic objects the data are rather abundant and fairly well known. However the values for groups and clusters of galaxies are rather poorly understood, as remarked also by Shull et al. (2012). We spend a few words on this issue (see also last part of lecture III) based principally on the work by Vikhlinin et al. (2006), Gonzalez et al. (2007), Lin et al. (2012), see also Zibetti & Goldmine Research Team (2005). The understanding and measurement of the mass and mass profile in clusters of galaxies, mainly in relation to the missing mass or DM problem, has been a search that started in the early 70 (see Chincarini

0.022, $H_0 = 67.3$, $\Omega_\mathrm{m} h^2 = 0.14$, $\Omega_\mathrm{b} = 0.049$, $f_\mathrm{b} = \Omega_\mathrm{b} h^2 / \Omega_\mathrm{m} h^2 = 0.157$.

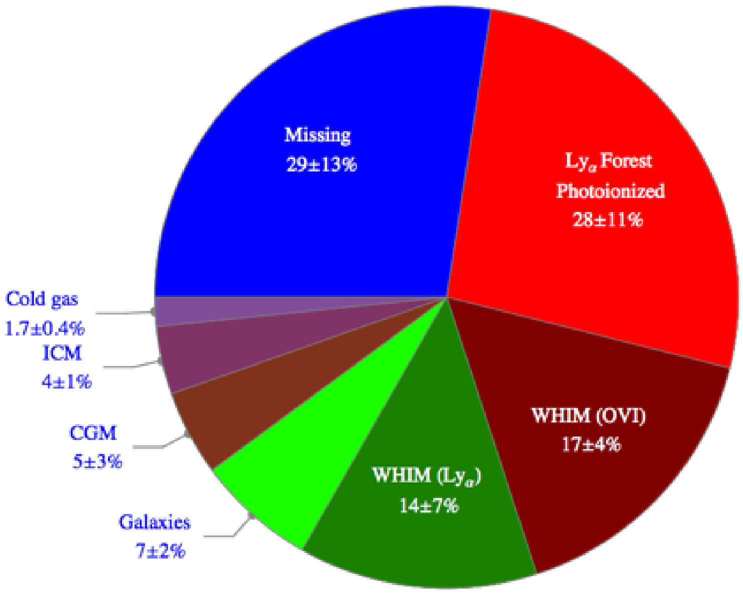

Fig. 2: Compilation of current observational measurements of the low redshift baryon census. Slices of the pie chart show baryons in collapsed form, in the circumgalactic medium (CGM) and intercluster medium (ICM), and in cold gas (HI and HeI). Primary baryon reservoirs include diffuse photoionized $Ly\alpha$ forest and WHIM traced by OVI and broad $Ly\alpha$ absorbers. BLAs and OVI have a combined total of $25\% \pm 8\%$, smaller than their direct sum (17% plus 14%) owing to corrections for double-counting of WHIM at $10^5 - 10^6$ K with detectable metal ions. Collapsed phases (galaxies, CGM, ICM, cold neutral gas) total $18\% \pm 4\%$. Formally, $29\%^\circ \pm 13\%$ of the baryons remain unaccounted for. Simulations suggest that an additional 15% reside in X-ray-absorbing gas at $T \geq 10^{6.3}$ K. Additional baryons may be found in weaker lines of low-column-density OVI and $Ly\alpha$ absorbers. Deeper spectroscopic UV and X-ray surveys are desirable. From Shull et al. (2012), see the paper for details.

(2013) for an account of the early 70s) and is still proceeding. The luminosity of galaxies has been highly improved in accuracy, the observations and measurements of the X-ray emission by the ionized gas detected to R_{500} and R_{200}. This allowed both accurate estimate of the mass, mass profile and of the physical parameters of the gas and finally now we have good estimates of the intra-cluster light (ICL). That intra-cluster light had likely to be in cluster it was discussed by many and however to my recollection the first to measure it in the Coma Cluster was de Vaucouleurs & de Vaucouleurs (1970). He realized indeed that that light might represent a large contribution to the cluster mass. On the other hand he finally interpreted it not as due to a diffuse component following the cluster potential but rather as due to the outermost corona of the central bright galaxies. Further work on the diffuse light in the Coma Cluster is due to Gunn & Melnick (1975) and Melnick et al. (1977). The smooth intergalactic background, according to the authors, follows the distribution of galaxies. This is quite reasonable since assuming they are stars that due to various interactions among galaxies, tidal interactions especially, have been detached from the galaxies, they would follow the cluster potential. White (1978) had already

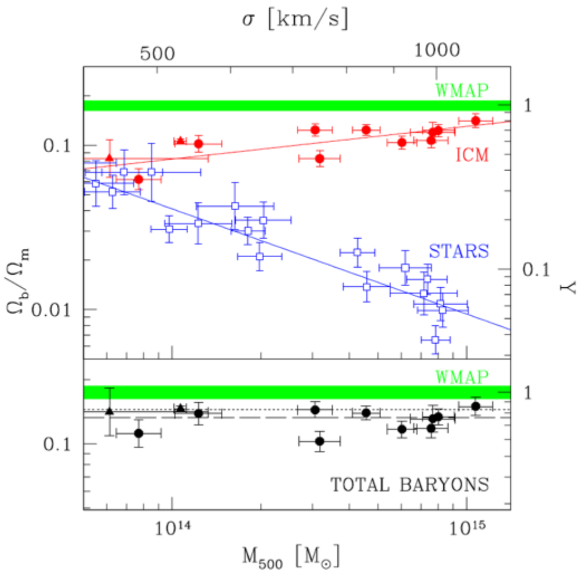

Fig. 3: Gonzales et al. (2007, 2013, 2015).

done simulations related to the Coma cluster and concludes that the cluster must be bound by intergalactic material. Melnick et al. (1977) conclude that the closure hypothesis for the cluster would call for stars with masses less than $0.1M\odot$ with $M/L = 700$.

Jumping to recent years Vikhlinin et al. (2006) find that the gas fraction within R_{2500} are significantly lower that the values expected from the SBBN & CMB and that however is getting closer to the universal value for $R > R_{2500}$. Gonzalez et al. (2007) estimate the contribution to the mass using the galaxies, the intra-cluster stars (ICL) and the gas. They find that within R_{500} and R_{200} the Bright Cluster Galaxies and the intra-cluster light contribute respectively 40% and 33% of the total stellar light. I reproduce here they rather significant plot, Fig. 3. Most important as far as our topics is concerned, Gonzalez et al. (2007) - see also Lin et al. (2012)) they do not observe in the baryon fraction any trend with the cluster mass and the observed total mass fraction within R_{500} is constant for systems with mass $6 \times 10^{13} M_{\odot} < M_{\rm cluster} < 10^{15} M_{\odot}$. Improved analysis (Gonzalez et al. (2013) Gonzalez 2015) gives $f_{\rm bary} \propto M_{500}^{0.16\pm0.04}$. For masses $M_{500} > 2 \times 10^{14}$ the baryonic fraction is only 7% below the value derived by the Planck team. We are getting close to have all the baryons we need at least at these scale lengths. Finally Giodini (2009) find that the baryonic fraction increases with mass according to the following relation: $f_{500}^{\rm stars+gas} = (0.123{\pm}0.003)\left(\frac{M_{500}}{2{\times}10^{14}}\right)^{0.09\pm0.03}$. The cluster results are very robust and there is good agreement among the various estimates. McGaugh et al. (2010) review the baryonic fraction on different scales, Fig. 4. The baryonic mass is plotted as a function of a velocity that has been derived either accounting for the over density of the cluster, M_{500} is transformed in V_{500}, or the observed rotational velocity of galaxies. In this way they plot the baryonic mass as a function of the mass of

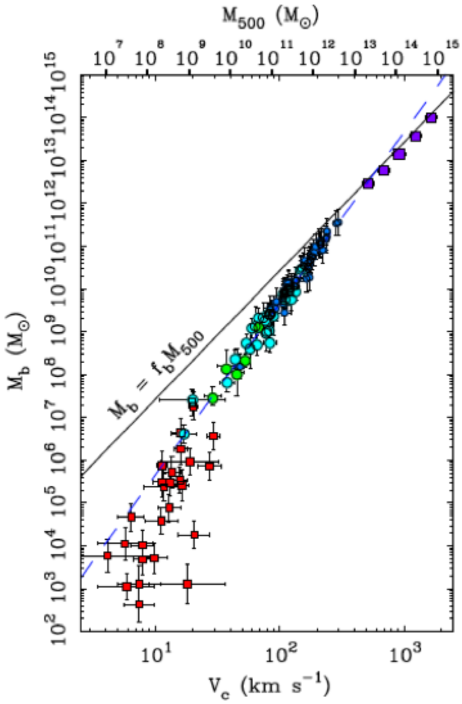

Fig. 4: Relation between baryonic mass and rotation velocity, McGaugh et al. (2010).

the systems and evidences different regimes in the correlation baryonic mass versus dynamical mass. Baryons are largely missing on small scales and, as we have seen, tend to match the cosmic abundance on larger scales. If all systems formed with the cosmic baryonic fraction, a reasonable hypothesis, it looks like massive systems were capable to keep their baryons while gradually lighter systems loosed (or transformed their status) them during their evolution. A working hypothesis could be that violent events occurred at some time in most cosmic objects and that high temperature gas escaped in those object where the potential well is shallow.

On galactic scale it is quite challenging the search that various team are carrying out to detect highly ionized material (Warm Hot Intergalactic Medium - WHIM). Here we refer for simplicity to the paper by Nicastro et al. (2016) - however see references therein - where it is stated that the baryon discrepancy for the MW, if we account for the detection of a hot halo of ionized material (Circumgalactic Medium) that has been identified using X ray observations of the OVII line K_α and K_β , its mass is estimated to be sufficient to finally close the Galaxy's baryon census. In this case the baryonic Mass would be about $M_\mathrm{b} = (0.8 - 4) \times 10^{11} M_\odot$. The proposed distribution, following model fitting, is that of an halo with a bubble in its center of about 6 kpc radius. The temperature, characteristic of the OVII lines, is about $T \sim 0.6 - 1 \times 10^6$ K. The errors quoted are still very large given the weakness of the lines, on the other hand this is a strong indication that is corroborated by various observations. The fascinating matter is that such a model, hopefully it will be possible to get higher S/N data, would agree with the bubbles observed by Planck,

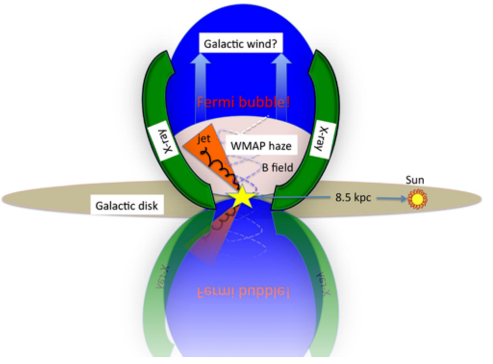

Fig. 5: From Su et al. (2010).

WMAP and Fermi (Dobler et al., 2010; Su et al., 2010). A sketch of the model is given in Fig. 5. The two blue bubbles symmetric to the Galactic disk indicate the geometry of the gamma-ray bubbles observed by the Fermi-LAT. The green arcs refer to soft X ray observations by ROSAT. The pink regions map the Radio observations (WMAP in this case). The electrons are at extremely high energy and reach a few hundreds GeV. It is out of the scope to describe the details of these observations and the models, but it was necessary to call attention to the hardness in detecting, and the detection of, baryonic matter in this state.

The search for DM particles

Now that we know rather well the distribution of matter, how much Dark Matter there is in the Universe and in part its distribution, the question is how do we plan the search for such particles? Any search must be based on the knowledge of what we are looking for or, if not the knowledge, at least based on some indication about mass, cross section or whatever guideline the theory tell us. As we all know so far we didn't get much in spite of the huge amount of resources used to this end. On the other hand it is very interesting to briefly discuss why we searched for Weak Interacting Massive Particles (WIMP) and Axions delineating at first the problem.

There is no doubt, indeed, that only the detection of these eluding particles will settle the matter, we will then have the tools for a deeper understanding of the Cosmo and of how it came about. However, as we all know, the CERN Large Hadronic Collider, (LHC) didn't yet detect any particle, nor did the various experiments came up yet with a certain and uncontroversial answer (see DAMA experiment however). The no-show (no detection) have led physicists to turn to the Dark Sector and less expensive high precision experiments, Science (September 2017). The fact that no particle has been yet detected is likely an indication, assuming it exists, of a very small cross section as to make it difficult to react with the detector or a strong indication that we are doing something wrong. It is reasonable to assume that these particles (they cannot come out of nothing) must have been part of the primordial Yelm from which it froze out at some cosmic time, at a time when the temperature of the Yelm was of the order of its mass. See Fig. 6 for a schematic

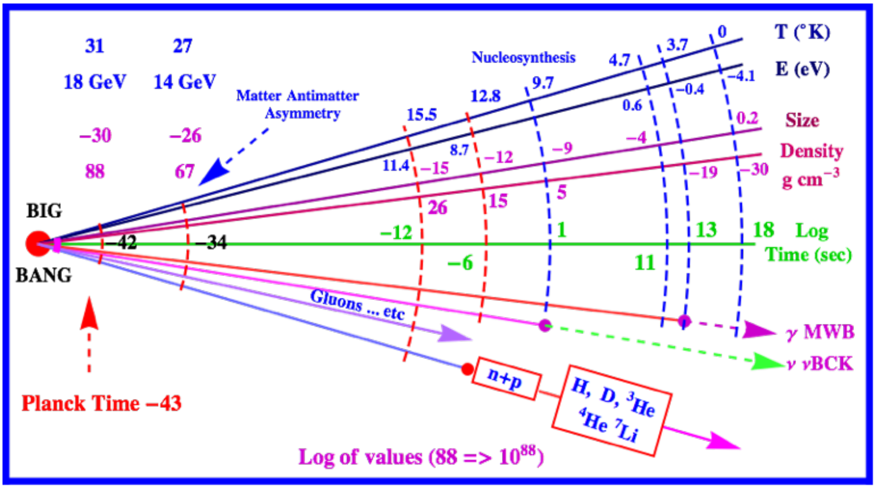

Fig. 6: Brief summary of the thermal history of the Universe, from the Planck time ($t \sim 10^{-43}$ & $T \sim 10^{19}$ GeV $\sim 10^{32}$ K) to the present. Primordial nucleosynthesis follows when $t \sim 10^{-2}$ to 10^2 and $T \sim 10$ to 0.1 MeV. Neutrinos species decouple from the plasma at temperatures of about 1 MeV and generate a background at $T = 1.96$ K. At $T \sim 3575$ K (0.308 eV) we have recombination, the baryon to photons ratio $\eta = (\Omega_{\rm B} h^2) 2.68 10^{-8}$.

view of the history of the thermal Universe. And the big question remains: How the Universe decides which particles to create? We apply, to have some indications on the nature of these particles, computing procedures with which we are somewhat familiar, the procedure that is similar to what we do in relation to the estimate of the neutron - proton ratio and the formation of Hydrogen at recombination. The key of the computation is the Boltzmann equation for annihilation. This problem is dealt with, among may others, by Kolb & Turner (1990) in their excellent books, by Dodelson (2008) Dodelson (2003), by the Berkeley lectures of Murayama (2007) and by Flip Tanedo's excellent article. We will use, as in the above references, natural units.

Note on Natural Units:

Physicists in general prefer to use natural units and some text books, as for instance Padmanabhan's "Structure formation in the Universe" , use as a basis such units. An easy way to transform from cgs to natural units (and vice-versa) , where energy in eV is the reference unit and $\hbar = k = c = 1$ is to use dimensions. The values of these constants in cgs are:

$$1c = 2.99792458 \times 10^{10}\,\mathrm{cm\,s^{-1}},$$

$$1\hbar = 1.05457266(63) \times 10^{-27}\,\mathrm{g\,cm^2\,s^{-1}},$$

$$1eV = 1.60217733(49) \times 10^{-12}\,\mathrm{g\,cm^2\,s^{-2}}.$$

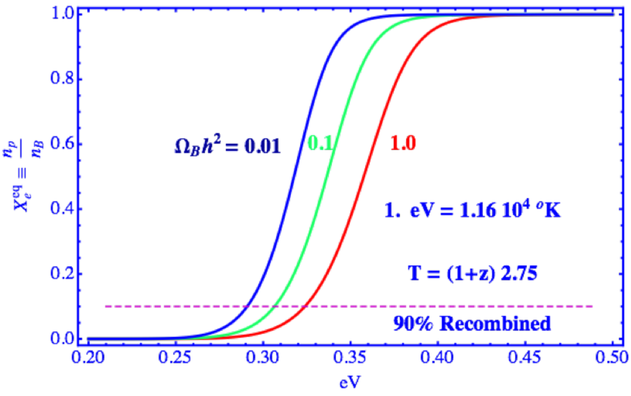

Fig. 7: Running of the ionisation as a function of time for $\Omega_{\rm B}h^2 = 0.01, 0.1, 1$. Time is increasing from right to left, temperature from left to right.

Example:

$$1\mathrm{GeV}^{-1} = \frac{1}{\mathrm{GeV}} = \frac{\mathrm{g\,cm^2\,s^{-1}}}{\mathrm{g\,cm^2\,s^{-2}}}\,\mathrm{cm\,s^{-1}} = \frac{1.05\times10^{-27}}{10^9\,1.6\times10^{-12}}\,2.998{\times}10^{10} = 1.97{\times}10^{-14}\,\mathrm{cm},$$

$$1\mathrm{GeV}^{-1} = \frac{1}{\mathrm{GeV}} = \frac{\mathrm{g\,cm^2\,s^{-1}}}{\mathrm{g\,cm^2\,s^{-2}}} = \frac{1.05\times10^{-27}}{10^9\,1.6\times10^{-12}} = 6.56\times10^{-25}.$$

End Note

But let's first go over the Saha law. Saha deals with a reaction that is in equilibrium, in particular if we refer to the recombination of protons and electrons, $p+e \to H+\gamma$, we have for a gas that remains neutral:

$$\begin{gathered}
p + e \to H + \gamma,\\
\text{neutrality } n_{\rm p} = n_{\rm e}, \quad n_{\rm H},\, n_{\rm p},\, n_{\rm e},\, n_{\rm B} = n_{\rm H} + n_{\rm p},\\
n_i = g\left(\tfrac{m_i T}{2\pi}\right)^{3/2} e^{\frac{\mu_i - m_i}{eT}} \begin{cases} i = \\ e, p, H \end{cases} \& \ \mu_{\rm p} + \mu_{\rm e} = \mu_{\rm H} \ \&\ m_{\rm H} \sim m_{\rm p},\\
\frac{n_{\rm H}}{n_{\rm p} n_{\rm e}} = \frac{g_{\rm H}}{g_{\rm e} g_{\rm p}}\left(\frac{m_{\rm H}}{m_{\rm e} m_{\rm p}}\frac{2\pi}{T}\right)^{3/2},\\
\frac{n_{\rm H}}{n_{\rm p} n_{\rm e}} = \frac{g_{\rm H}}{g_{\rm e} g_{\rm p}}\left(\frac{m_{\rm H}}{m_{\rm e} m_{\rm p}}\frac{2\pi}{T}\right)^{3/2} e^{\left(\frac{B = m_{\rm p} + m_{\rm e} - m_{\rm H} = 13.6\ \mathrm{eV}}{T}\right)} =\\
= \frac{g_{\rm H}}{g_{\rm e} g_{\rm p}}\left(\tfrac{m_{\rm e} T}{2\pi}\right)^{-3/2} e^{B/T},\\
X_{\rm e} = \frac{n_{\rm p}}{n_{\rm B} = n_{\rm p} + n_{\rm H}}\ g_{\rm e} = 2\ g_{\rm p} = 2\ g_{\rm H} = 4,\\
\frac{1 - X_{\rm e}^{\rm eq}}{(X_{\rm e}^{\rm eq})} = \frac{4\sqrt{2}\xi(3)}{\sqrt{\pi}}\left(\frac{T}{m_{\rm e} = 0.51\ \mathrm{MeV}}\right)^{3/2} \eta e^{B/T} \begin{cases} \eta = (\Omega_{\rm B} h^2) 2.68\times10^{-8},\\ T = (1+z)2.75\ \mathrm{K}. \end{cases}
\end{gathered}$$

With the temperature decreasing, Fig. 7, most of the electrons and protons recombine and the abundance of free particles goes to zero. It is generally assumed that the temperature of recombination is reached when $X_{\rm e} = 0.1$. With the decrease of the temperature (increase of time and decrease of the density) in an expanding Universe the probability that a proton combine with an electron decreases considerably and at some point the process stop. In other words the process ends when the expansion rate is larger than the reaction rate: $\Gamma \leq H_{\mathrm{Hubble}}$. Using this criterium we have:

$$\begin{gathered}
\sigma_{\mathrm{T}} = 6.65 \times 10^{-25}\mathrm{cm}^2, \\
\Gamma = n_{\mathrm{p}} \langle \sigma v \rangle = (X_{\mathrm{e}} \eta n_\gamma) \langle \sigma v \rangle, \text{ since we have here} \\
X_{\mathrm{e}}^{\mathrm{eq}} \ll 1 \; X_{\mathrm{e}}^{\mathrm{eq}} \approx 0.51 \eta^{-1/2} \left(\tfrac{m_{\mathrm{e}}}{T}\right)^{3/4} \exp(-B/T), \\
n_\gamma = \tfrac{30\xi(3)\alpha T^3}{\pi^4 k_{\mathrm{B}}} \begin{cases} 20.3883 T_{\mathrm{K}}, \\ 3.18 \times 10^{13} T_{\mathrm{eV}}, \end{cases} \\
\eta = 2.68 \times 10^{-8} (\Omega_{\mathrm{B}} h^2) \;\; \langle \sigma v \rangle_{\text{Thermally averaged}} \cong 4.7 \times 10^{-24} \left(\tfrac{T}{1\,\mathrm{eV}}\right)^{-1/2} \mathrm{cm}^2.
\end{gathered}$$

From the Friedman equations for the matter dominated era:

$$\begin{gathered}
H_0 = 100\, h \text{ km s}^{-1} \text{ Mpc} = 2.13 \times 10^{-42} \text{ GeV} = 2.13 \times 10^{-33} \text{ eV}, \\
H_0 = \tfrac{2.13 \times 10^{-33} 1.6 \times 10^{-12}}{1.05 \times 10^{-27} 2.998 \times 10^{10}} = 1.08 \times 10^{-28}, \\
H(\equiv H_{\mathrm{Hubble}}) = H_0 \sqrt{\Omega_{\mathrm{m}} \left(\tfrac{T^3}{T^3_{\gamma,0}}\right)} = \\
= 1.08 \times 10^{-28} \left(\tfrac{2.7}{1.05 \times 10^4}\right)^{3/2} (\Omega_{\mathrm{m}} h^2)^{1/2} \left(\tfrac{T}{1\,\mathrm{eV}}\right)^{3/2} = \\
= 3 \times 10^{-23} (\Omega_{\mathrm{m}} h^2)^{1/2} \left(\tfrac{T}{1\,\mathrm{eV}}\right)^{3/2} \mathrm{cm}^{-1}.
\end{gathered}$$

It follows from equating the reaction rate with the expansion rate:

$$\begin{gathered}
H = 3 \times 10^{-23} \text{ cm}^{-1} (\Omega h^2)^{1/2} T_{\mathrm{eV}}^{3/2}, \\
\text{from } \Gamma = H \\
T_{\mathrm{eV}}^{-1/4} e^{6.8/T_{\mathrm{eV}}} = 8.08 \times 10^{12} \left(\tfrac{\Omega_{\mathrm{B}}}{\Omega}\right), \;\; \text{I solve it graphically} \\
T_{\mathrm{eV}} = 0.23 \text{ eV} \\
X_{\mathrm{e}}(0.23) = 2.6 \times 10^{-5}.
\end{gathered}$$

Kolb and Turner, using a more accurate derivation, estimate $T_{\mathrm{eV}} = 0.26$ eV.

A gas of weakly interacting particles with g internal degrees of freedom is characterised by its distribution - occupancy function in the phase space. The density of particles, the energy density and the pressure are (for the i species in equilibrium – eq):

$$\begin{gathered}
n_{i,\mathrm{eq}} = \tfrac{g}{(2\pi)^3} \int f(\overline{p}) d^3p, \\
\rho_{i,\mathrm{eq}} = \tfrac{g}{(2\pi)^3} \int E(\overline{p}) f(\overline{p}) d^3p, \\
p_{i,\mathrm{eq}} = \tfrac{g}{(2\pi)^3} \int \tfrac{|\overline{p}|^2}{3E} f(\overline{p}) d^3p,
\end{gathered}$$

since $E^2 = |\overline{p}|^2 + m^2$ [3], $dp = \frac{EdE}{p}$; using spherical coordinates $p^2 = p_{\mathrm{x}}^2 + p_{\mathrm{y}}^2 + p_{\mathrm{z}}^2$ with $p_{\mathrm{x}} = p \sin\theta \cos\varphi$, $p_{\mathrm{y}} = p \sin\theta \sin\varphi$, $p_{\mathrm{z}} = p\cos\theta$, $d^3p = 4\pi p^2 dp = 4\pi(E^2 - m^2)^{1/2} EdE$.

The above equations can be written as:

$$n = \frac{g}{2\pi^2} \int \sqrt{E^2 - m^2} f(\overline{p}) EdE,$$

$$\rho = \frac{g}{2\pi^2} \int \sqrt{E^2 - m^2} f(\overline{p}) E^2 dE,$$

[3] Natural units as in: Kolb & Turner (1990) Kolb and Turner (1994), Dodelson (2008) Dodelson (2008). For a derivation see also the Lecture by Kip Thorne at Caltech (Kinetic section 3.5.3). In cgs units $E^2/c^2 = p^2 + m^2c^2$.

$$p = \frac{g}{6\pi^2}\int (E^2 - m^2)^{3/2} f(\overline{p}) dE,$$

and the distribution function (with $+1$ for Fermi-Dirac species and -1 for Bose - Einstein species):

$$f(\overline{p}) = \frac{1}{Exp^{E-\mu}/T^{\pm 1}}.$$

In the non relativistic limit ($m \gg T$) the number density for a given species:

$$\begin{aligned}
n &= \frac{g}{2\pi^2}\int \sqrt{E^2 - m^2} f(\overline{p}) E dE \\
&= \frac{g}{2\pi^2}\int \frac{\sqrt{E^2 - m^2} E dE}{\exp\left[\frac{E-\mu-E+m}{T}\right]} \exp\left[-\frac{E-m}{T}\right] \\
&= \frac{g}{2\pi^2}\exp\left[-\frac{E-m}{T}\right]\int p^2 dp \exp\left[-\frac{p^2}{2mT}\right] \Rightarrow \text{ set } p^2 = x \\
&= \frac{g}{2\pi^2}\exp\left[-\frac{E-m}{T}\right]\int \frac{1}{2}x^{\frac{1}{2}} e^{\frac{x}{2mT}} = g\left(\frac{mT}{2\pi}\right)^{3/2} \exp\left[-\frac{E-m}{T}\right],
\end{aligned}$$

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-\frac{m}{T}} = gT^3\left(\frac{m}{2\pi T}\right)^{3/2} e^{-\frac{m}{T}}.$$

For relativistic particles (in this case to make the reader familiar with unit systems we use CGS units), bosons, where $g = 2$, $m = 0$ setting $x = \frac{|p|c}{kT}$, $E = \left[(pc)^2 + (mc^2)^2\right]^{1/2}$

$$n = \frac{g}{2\pi^2}\left(\frac{kT}{\hbar c}\right)^3 \int_0^\infty \frac{x^2 dx}{e^{\frac{E-\mu}{kT}} \pm 1} = \frac{g}{2\pi^2}\left(\frac{kT}{\hbar c}\right)^3 \int_0^\infty \frac{x^2 dx}{e^{\frac{pc-\mu}{kT}} \pm 1},$$

$$n = \frac{g}{2\pi^2}\left(\frac{kT}{\hbar c}\right)^3 \int_0^\infty \frac{x^2 dx}{e^{-x} \pm 1} = \frac{g}{2\pi^2}\left(\frac{kT}{\hbar c}\right)^3 2\varsigma(3) = \frac{2.404}{\pi^2}\left(\frac{kT}{\hbar c}\right),$$

where $\varsigma(3)$ is the Riemann's zeta function. The Boltzman equation for species 1 and 2 with the reaction products 3 and 4 in an expanding Universe is written as:

$$a^{-3}\frac{d}{dt}n_1 a^3 = n_{1,\mathrm{eq}} n_{2,\mathrm{eq}} \langle\sigma\nu\rangle \left\{\frac{n_3 n_4}{n_{3,\mathrm{eq}} n_{4,\mathrm{eq}}} - \frac{n_1 n_2}{n_{1,\mathrm{eq}} n_{2,\mathrm{eq}}}\right\}.$$

To better understand the significance of the equation related to the time at which some particles freeze and how these relics determine the abundances we measure nowadays, let's derive first the equation conceptually (for details see Steigman, 1979).

We consider a system containing N particles (antiparticles) in a volume V with n particles per unit volume ($N = nV$). In the radiation dominated phase the dependence of the volume and time from the Temperature is $V \propto T^{-3}$ and $t \propto T^{-2}$. We assume furthermore that $n = \overline{n}$. Particles are produced at a rate ψ and annihilate βn^2, so that with the running of time we have

$$\frac{dN}{dt} = \psi V - \beta n^2 V.$$

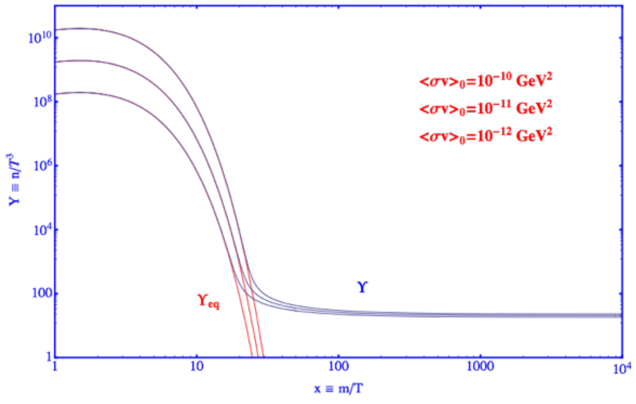

Fig. 8: Abundance of stable particles as a function of the temperature. The red line refers to the abundance of particles in equilibrium. When the mass to temperature ratio is about $10-30$ particles break from equilibrium and we are at about the time of freeze out, $x_\mathrm{f} = 10-30$.

The equilibrium condition $\frac{dN}{dt} = 0$ leads to $\psi = \beta n_\mathrm{eq}^2$, ($n_\mathrm{eq}(T)$ is the density of particles i at the temperature T) and we can write the above equation as

$$\frac{dN}{dt} = \beta(T)n_\mathrm{eq}^2 V - \beta(T)n^2 V = \beta(T)(n_\mathrm{eq}+n)(N_\mathrm{eq}-N).$$

It is convenient, instead of using the temperature T, to define $x(t) \equiv \frac{mc^2}{kT} = \frac{m(\mathrm{GeV})}{T(\mathrm{GeV})}$. At high temperatures $x \leq 1$ the reactions (production and annihilation) go both ways and we have equilibrium, $N = N - eq$. However with the decrease of the temperature $x \geq 1$ we have a deviation from equilibrium that we could characterise as $\Delta = \frac{N-N_\mathrm{eq}}{N_\mathrm{eq}}$. When the temperature decreases the number of particles produced decreases and it is matter to define a threshold, for instance $\Delta_* = 1/2$, that determines the value of $x = x_*$, so that for $x \leq x_*$ we have equilibrium $N \approx N_\mathrm{eq}$, while for $x \geq x_*$ annihilation dominate. At some point however annihilation will stop. As stated earlier, with the density of the Universe decreasing the particles that have been left do not interact anymore and they become the relics of the reaction, the abundance has been frozen. The left hand side of the Boltzmann equation $a^{-3}\frac{d}{dt}n_1 a^3 = n_{1,\mathrm{eq}}n_{2,\mathrm{eq}}\langle\sigma\nu\rangle\{\frac{n_3 n_4}{n_{3,\mathrm{eq}}n_{4,\mathrm{eq}}} - \frac{n_1 n_2}{n_{1,\mathrm{eq}}n_{2,\mathrm{eq}}}\}$ is of the order $\frac{n_1}{t} \sim n_1 H$. With a reaction rate $n_2\langle\sigma\nu\rangle$ larger than the expansion rate and with the second term being of the order $n_1 n_2\langle\sigma\nu\rangle$, the only way to maintain equality is for the terms on the right to cancel out (that is to say that in equilibrium we have $a^{-3}\frac{d}{dt}n_1 a^3 = 0$).

$$\frac{n_3 n_4}{n_{3,\mathrm{eq}}n_{4,\mathrm{eq}}} = \frac{n_1 n_2}{n_{1,\mathrm{eq}}n_{2,\mathrm{eq}}},$$

and if we are dealing with electrons and protons combining to form Hydrogen, recombination, we have the well known Saha equation: $p + e \to H + \gamma$ and from the above: $\frac{n_\mathrm{H}}{n_{\mathrm{H,eq}}} = \frac{n_\mathrm{e}n_\mathrm{p}}{n_{\mathrm{e,eq}}n_{\mathrm{p,eq}}} \Rightarrow \frac{n_\mathrm{e}n_\mathrm{p}}{n_\mathrm{H}} = \frac{n_{\mathrm{e,eq}}n_{\mathrm{p,eq}}}{n_{\mathrm{H,eq}}}$ X_e = etc.... (see also presentation). As discussed above we have a time (Temperature) of transition between the period of equilibrium (high T) and the time annihilation dominates. Since this depends on the energy (mass) of the particles involved the dividing line must be at $\frac{m}{T} \sim 1$. Two heavy particles (X) interact and produce two very light particles that couple very strongly with the cosmic plasma. In this case Weakly Interacting Massive Particles (WIMP)., we can write ($n_1 \equiv n_{\mathrm{X}}, n_2 \equiv n_{\mathrm{X}}, n_3 \equiv n_l$ and $n_4 \equiv n_{\mathrm{l}}$ with $n_{\mathrm{l}} = n_{\mathrm{l,eq}}$)[4].

$$a^{-3}\frac{d}{dt}n_1 a^3 = n_{1,\mathrm{eq}} n_{2,\mathrm{eq}} \langle\sigma\nu\rangle \{1 - \frac{n_1 n_2}{n_{1,\mathrm{eq}} n_{2,\mathrm{eq}}}\},$$

$$a^{-3}\frac{d}{dt}n_{\mathrm{X}} a^3 = \langle\sigma\nu\rangle \{n_{i,\mathrm{eq}}^2 - n_{\mathrm{X}}^2\}.$$

The products of the reaction are coupled to the plasma and under the reasonable assumption that particles and antiparticles are in the same number, indeed the equality is satisfied in our quasi baryon - symmetric[5], Universe with an accuracy of 10^{-10}, we can derive the abundance n_X of the heavy particles (Fig. 8).

Defining $\Upsilon \equiv \frac{n_{\mathrm{X}}}{T^3}$ (Kolb and Turner define $\Upsilon \equiv \frac{n_{\mathrm{X}}}{s}$ where s is the entropy $s \propto T^3$ so that the difference is only in a conversion factor). We furthermore define $x = \frac{m}{T}\left(i.e. \frac{mc^2}{kT}\right)$ and recall that $(a^3T^3) = \mathrm{const}$,

$$\frac{d}{dt}\Upsilon = T^3 \langle\sigma\nu\rangle \{\Upsilon_{\mathrm{eq}}^2 - \Upsilon^2\},$$

$$\frac{d}{dt}x = m\frac{1}{T^2}\frac{dT}{dt} = m\frac{1}{T^2}\frac{1}{a}\frac{\dot{a}}{a} = m\frac{1}{T^2}HT = Hx,$$

$$H_{\mathrm{m}} = H(m = T) \quad \frac{H}{H_{\mathrm{m}}} = \frac{\rho^{1/2}}{\rho_{\mathrm{m}}^{1/2}} = \frac{T^2}{T_{\mathrm{m}}^2} = \frac{T^2}{m^2}.$$

Defining $\lambda \equiv \frac{m^3\langle\sigma\nu\rangle}{H_{\mathrm{m}}}$ we can write $\frac{d\Upsilon}{dx} = \frac{m^3}{H_{\mathrm{m}}x^2}\langle\sigma\nu\rangle \{\Upsilon^2 - \Upsilon_{\mathrm{eq}}^2\}$, and finally derive

$$\frac{d\Upsilon}{dx} = -\frac{\lambda}{x^2}\{\Upsilon^2 - \Upsilon_{\mathrm{eq}}^2\}.$$

The equation (a form of the Riccati equation) can be solved only numerically. The annihilation rate goes as the equilibrium density times the thermally averaged annihilation cross section

$$\Gamma_{\mathrm{A}} \propto n_{\mathrm{eq}} \langle\sigma\nu\rangle \begin{cases} n_{\mathrm{eq}} \propto T^3 \quad \text{relativistic,} \\ n_{\mathrm{eq}} \propto (mT)^{3/2}e^{-m/T} \quad \text{nonrelativistic,} \end{cases}$$

and in both cases the cross section decreases with T so that at some point it becomes impotent, roughly at $\Gamma_{\mathrm{A}} \simeq H$ and we can define $x = x_{\mathrm{f}}$ the moment at which this "freeze out" occurs. We expects that Y tends to follow Y_{eq} for small x while after freeze out ($x \gg x_{\mathrm{f}}$) Y_{eq} decay exponentially and $Y \gg Y_{\mathrm{eq}}$. That is for $x \Rightarrow \infty$ we have

$$\frac{d\Upsilon}{dx} \approx -\frac{\lambda(x)}{x^2}\Upsilon^2,$$

and now the WIMP are not relativistic. Solving the differential equation (use Mathematica for instance) we have: $\Upsilon(x) = \frac{x}{\lambda + xC_1}$ and for $x \to \infty$, $C_1 = -\frac{1}{\Upsilon_\infty}$, therefore

[4]For a rigorous derivation of the X WIMP equation see Kolb & Turner (1990).

[5]The baryon symmetry is briefly discussed in the Appendix to this section

at $x = x_{\rm f}$, we have $\frac{1}{\Upsilon_\infty} - \frac{1}{\Upsilon_{\rm f}} = \frac{\lambda}{x_{\rm f}}$, where $\Upsilon_{\rm f} = \Upsilon(x_{\rm f})$. Since $\Upsilon_{\rm f} \gg \Upsilon_\infty$, we have the approximate relation $\frac{1}{\Upsilon_\infty} \simeq \frac{\lambda}{x_{\rm f}}$, of course an accurate solution is also feasible (see Kolb & Turner, 1990).

At this point we can use the observed value $\Omega_{\rm DM}$ to constrain the cross section. In the evolving Universe generally $(at)^3 = $ const. However when we have annihilation with generation of photons or other relativistic particles the quantity is not conserved and it is as if the decrease of temperature slow down. We can define a quantity $s = T^{-1}(\rho + p)$, assuming $\mu \ll T$ in the following equation derived from Boltzmann equations above

$$d(sa^3) \equiv d\left\{\frac{a^3}{T}(\rho + p - n\mu)\right\} = \left(\frac{\mu}{T}\right) d(na^3).$$

In all cases of interest we will have either $(na^3) = $ const or $\mu \ll T$, so that the quantity sa^3 is conserved. I we write the above equation, always for $\mu \ll T$, as $Td(sa^3) = d(\rho a^3) + pd(a^3)$, we notice it is very similar to $TdS = dE + pdV$, and the amount $s = T^{-1}(\rho + p)$ can be interpreted as the entropy density of the system. To compute the Entropy we simply integrate the energy density previously given in the approximation $T \gg m$ and $T \gg \mu$

$$\rho = \frac{g}{2\pi^2}\int_0^\infty \frac{E^3 dE}{e^{E/kT} \pm 1} = \begin{cases} g_{\rm B}(\pi^2/30)T^4, \\ \frac{7}{8} g_{\rm F}(\pi^2/30)T^4, \end{cases}$$

and from the equation of state $p = 1/3\rho$, for the same temperature of boson and fermions we have $s = (g_{\rm B}{+}7/8g_{\rm F})\frac{2\pi^2}{45}T^3$, so that from $\left[(g_{\rm B} + 7/8g_{\rm F})\frac{2\pi^2}{45}T^3\right]_{T\sim 10\rm GeV} = \left[(g_{\rm B} + 7/8g_{\rm F})\frac{2\pi^2}{45}T^3\right]_{T_0}$, we derive $\frac{(aT)^3_{\sim 10\rm GeV}}{(aT)^3_{T_0}} = \frac{3.36}{91.5}$,

$$\begin{cases} g_{10\rm GeV} \Rightarrow \text{quarks} = \frac{7}{8}[2(p\overline{p}) * 5(\text{types}) * 3(\text{colors}) * 2(\text{spin})] = \frac{7}{860}, \\ \quad \Rightarrow \text{leptons} = \frac{7}{8}[2(p\overline{p}) * 6(e, \nu_e, \mu, \nu_\mu, \tau, \nu_\tau) * 2] = \frac{7}{8}24, \\ \quad \Rightarrow \text{photons} = 2, \\ \quad \Rightarrow \text{gluons} = 8(\text{colors}) * 2(\text{spin}), \\ \quad \frac{7}{8}(60 + 24) + 18 = 91.5, \\ \quad g_{T_0} \Rightarrow \text{photons} = 2, \\ \quad \Rightarrow \text{neutrinos} = \frac{7}{8} * 3 * 2 * (\frac{4}{11})^{4/3} = 1.36, \\ \quad 2 + 1.36 = 3.36. \end{cases}$$

Remember $\Upsilon = \frac{n_{\rm X}}{T^3}$, and the density therefore $\rho_{\rm X} = n_{\rm X} m = \frac{mn_{\rm X}}{T^3}T^3 = m\Upsilon T^3$. Finally we have $\Omega_{\rm X} = \frac{\rho_{\rm X}}{\rho_{\rm crit}} = \frac{m\Upsilon_\infty T_0^3(\frac{a_1 T_1}{a_0 T_0})^3}{\rho_{\rm crit}} = \frac{m\Upsilon_\infty T_0^3}{26\rho_{\rm crit}} = \frac{H_{\rm m} x_{\rm f} T_0^3}{m^2\langle\sigma\nu\rangle 26\rho_{\rm crit}}$. In the radiation era $H_{\rm m} = H(m = T) = (\frac{8\pi G\rho}{3})^{1/2}$, where $\rho = g_{\rm tot}\frac{\pi^2}{30}T^4 = g_{\rm tot}\frac{\pi^2}{30}m^4$, and putting all of this together

$$\Omega_{\rm X} = \frac{\left(\frac{8\pi G}{3} g_{\rm tot}\frac{\pi^2}{30}m^4\right) x_{\rm f} T_0^3}{m^2\langle\sigma\nu\rangle 26\rho_{\rm crit}} = \frac{\left(\frac{4\pi G}{45} g_{\rm tot}\right) x_{\rm f} T_0^3}{\langle\sigma\nu\rangle 26\rho_{\rm crit}}.$$

Following Dodelson (2008) and assuming all the Dark Matter particles are of the same nature and the temperature of interest for the DM particles production is of the order of 100 GeV we can write

$$\Omega_{\rm DM} = 0.3h^{-2}(\frac{x_{\rm f}}{10})(\frac{g_{\rm tot}}{100})^{1/2}\frac{10^{-39}\text{cm}^2}{\langle\sigma\nu\rangle}.$$

We have estimated a statistical weight of order of 100 close to what we estimated earlier. The theoretical literature on the characteristics of the various candidates and possibilities is very abundant. To our purpose we notice that the main characteristics are that the particles do not interact easily with the particles we know (of which the detector is made), the cross section is extremely low and the particles are stable otherwise there would be none nowadays. That their energy should be larger than $10 - 100$ GeV is a consequence of the fact they have not yet been detected in the accelerators experiments. Obviously the extremely small cross section is a fundamental indication for planning the experiments aimed to their detection.

Axions

Various authors proposed the solution of interacting particles with much smaller cross section, among these Spergel & Steinhardt (2000) Spergel and Steinhardt (1999), Hannestad & Scherrer (2000) Hannestad (1999) and Firmani et al. (2000) Firmani et al. (1999) who also suggested a cross section. For some astrophysical consequences of collisional DM see Ostriker (2000) Ostriker (1999).

Kolb & Turner (1990) in their book *The Early Universe* dedicate chapter 10 to the axions and illustrate very clearly the theory (they master the interplay between Astrophysics, Cosmology and particle physics) and the consequences the existence of axions would have in stellar evolution. Indeed stringent limits come from Astrophysics. Among the many recent and excellent papers and reviews that by Baudis (2016) is clear and concise.

AXIONS

The axion is a prediction made by Peccei and Quinn (PQ) in Quantum Chromodynamics. The mass of the axion proposed originally by PQ was of about 200 keV. This mass was ruled out and the mass has been taken essentially as a free parameter that could have any value in the range 10^{-12} eV to 1 MeV. Actions would act as a very efficient coolant so that for certain masses a strong emission of energy via axions would cause contraction of the star to raise the temperature and nuclear energy generation rate so that during this process the luminosity due to photons increases as well. The net effect would be the shortening of the star lifetime. A detailed analysis of stellar evolution and cosmology narrows the permissible mass window so that Kolb & Turner (1990) conclude that relic axions of mass 10^{-6} to 10^{-5} eV or 10^{-3} eV may well be the dark matter in the Universe!

Fig. 9 from Raffelt (2008) evidences the rejected windows from Astrophysics and Cosmology and the present days research window.

Note

The Universe in which we live, the one we see and feel, is simply due to the asymmetry left over from the annihilation of matter and antimatter. The question following the above analysis is why the asymmetry.

The reaction electrons positrons (Fermions) at temperatures of about 10^{12} degrees maintains equilibrium between $e\bar{e}$ and $\gamma\gamma$. Photons on the other hand with the decrease of the temperature are not conserved and the chemical potential con-

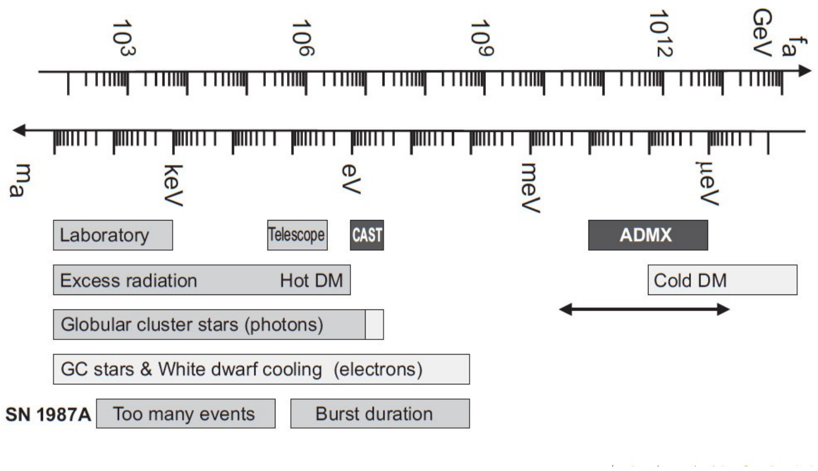

Fig. 9: Allowed windows of Axions energy from Astrophysics and Cosmology.

sequently is identical to zero so that because of the conservation of the chemical potential in the reaction we have $\mu_{\rm e} + \mu_{\overline{\rm e}} = 0$ or $\mu_{\rm e} = -\mu_{\overline{\rm e}}$, and the excess of electrons over positrons is

$$n - \overline{n} = \frac{g}{2\pi^2} \int \sqrt{E^2 - m^2} \left[\frac{1}{e^{(E-\mu)/T} + 1} - \frac{1}{e^{(E+\mu)/T} + 1} \right] E dE.$$

The solution of the integral is rather laborious and details on how to solve it are given in Landau and Lifshitz Statistical Physics Chapter V § 58 Page 169 in the 2007 Edition. For $T \gg m$

$$n - \overline{n} \simeq \frac{gT^3}{6\pi^2} \left[\pi^2 \left(\frac{\mu}{T} \right) - \left(\frac{\mu}{T} \right)^3 \right].$$

The number of photons (Bosons), chemical potential $\mu = 0$, $m = 0$ and statistical weight $= 0$ give $n_\gamma = \frac{g_\gamma}{2\pi^2} T^3 2\zeta(3)$, and the ratio excess of electron to photons is

$$\frac{n - \overline{n}}{n_\gamma} = \frac{g_{\rm e}}{g_\gamma} \frac{\pi^2}{6\zeta(3)} \left[\frac{\mu}{T} + \frac{1}{\pi^2} \left(\frac{\mu}{T} \right)^3 \right] = 10^{-8}.$$

The Universe appears to be electrically neutral so that the number of protons must be equal to the number of electrons that survived the annihilation, their ratio to photons is equal to $\frac{n_{\rm p}}{n_\gamma} = 10^{-8}$ as estimated for baryonic matter, see figure from Fields (2006), see also Kolb & Turner (1990).

If we apply the formalism described above for the freezing out of WIMP for a baryon symmetric Universe we would find a value of Y_∞ that is about 10 order of magnitude smaller than the observed value. The most reasonable explanation of the asymmetry (we live in a baryon asymmetric Universe) is that at $T \gtrsim 38$ MeV, the Universe possessed a number of baryons number of anti baryons asymmetry. This opens a new fascinating chapter: Baryogenesis.

EXPERIMENTS

As we have said the search for DM particles is one of the most fundamental task of modern Physics and Cosmology. In the following we will refer about some of the ongoing experiments designed to detect such particles. We have to keep in mind, however, that the hunt is open for an animal we know almost nothing about.

We, baryons, are host of a foreign Universe made of unknown matter that is by a large factor more abundant than the matter we know. In the visible Universe we have particles of various energies and mass so that it would seem strange that Nature decided to work in economy when creating the DM particles. In other words it is quite likely, I believe, that Nature created DM particles with various characteristics. It is a concern, however, that so far neither the high energy particle accelerators nor the experiments developed to detect this eluding particles were able to show they exist. Only one experiment, DAMA, is detecting since many years, a signal. The topics is still rather debated and the detection would need confirmation by some other experiment, but, no matter what, there is a signal. The coming years will clarify certainly this matter in details thanks to the many experiments DAMA-like nowadays planned.

The DAMA experiment, Bernabei et al. (2004)[6], bases the partial detection on the annual modulation signature as suggested by Drukier et al. (1986). The annual orbit of Earth about the Sun modulate the rotation velocity vector of the sun around the galaxy.

The solar motion relative to the Local Standard of Rest (LSR) is (from Binney and Tremaine) $U_\odot = 10.0\pm0.4\ \mathrm{km\,s^{-1}}$, $V_\odot = 5.2\pm0.6\ \mathrm{km\,s^{-1}}$, $W_\odot = 7.2\pm0.4\ \mathrm{km\,s^{-1}}$ a velocity of 13.4 $\mathrm{km\,s^{-1}}$ toward the Apex. The $x(U)$, $y(V)$ and $z(W)$ are set with x toward the Galactic Center, z toward the North Galactic Pole and the galactic rotation on the x–y plane counterclockwise as seen above the plane from the North. The mean velocity of Earth around the Sun is of 29.78 $\mathrm{km\,s^{-1}}$ while the Earth's rotational velocity of 0.465 $\mathrm{km\,s^{-1}}$. The inclination of the ecliptic on the Galactic Plane is $\gamma = 62\overset{\circ}{.}5$ so that the Earth velocity is

$$\nu_{\mathrm{E}} = \nu_\odot + \nu_\oplus \cos\gamma \cos\omega(t - t_0),$$

with $\omega = \frac{2\pi}{T}$, $T = 1$ year and $t_0 =$ June 2 ± 1.3 days according to the analysis by Griest (Fermi Lab report 1988 - unpublished). The velocity of the Sun is essentially the rotation of the LG around the Galactic Center since the component of the Sun relative to the LG is only of a few $\mathrm{km\,s^{-1}}$. In Fig. 10 in red the coordinates system while the black arrow indicate the motion of the Sun (Local Group) about the Galactic Center. The orbit of Earth (clearly the plot is not in scale) inclined of 62.5 degrees is shown with the location of the observer (blue dot). This modulation on the velocity of the detector causes a modulation in the number of particles with a maximum in June. Indeed in our journey trough the halo of the Galaxy the detector area A_0 is bombarded by particles that are viewed by it under a solid angle $d\Omega = \sin\theta d\theta d\phi = d\cos\theta d\phi$ A particle viewed under an angle θ will naturally see the detector area $A_0\cos\theta$. It is assumed that the particles are distributed in direction and momentum according to the function $f(p,\theta)$. Interacting with the detector

[6] The group of Rita Bernabei proposed the DAMA experiment in 1990 and since then the number of papers, presentations and proceedings has been copious and enlightening. Here I refer only to some papers that are useful, the student should look at the related literature however.

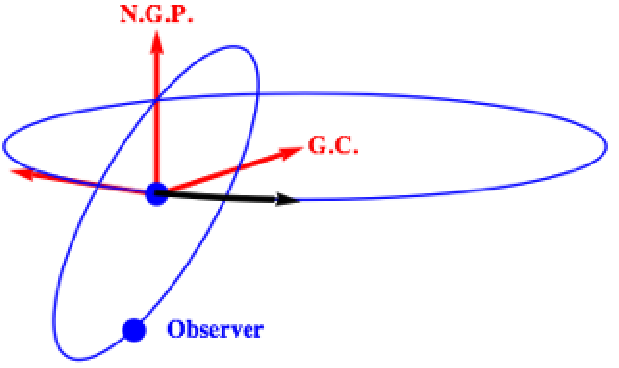

Fig. 10: The orbit of Earth referred to the Galactic coordinates.

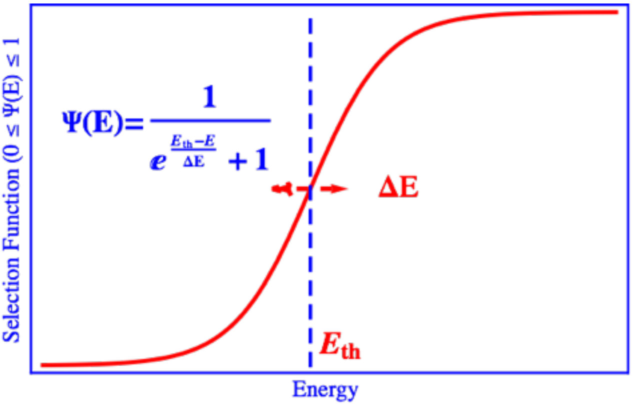

Fig. 11: Threshold curve of the detector accounting for the threshold width.

the DM particle generates a recoil particle that can be detected. The detector is obviously sensitive in a given range of energies and in particular it has a threshold beyond which there is no detection.

The width energy of the threshold $E_{\rm th}$ depends on the detector and we indicate it as ΔE. Particles with energies larger than $E_{\rm th} + \Delta E$ are detected, particles with Energy smaller than $E_{\rm th} - \Delta E$ are not detected and in between the probability of detection is a function of the Energy. A convenient analytical expression to represent this characteristics of the detector is given by the selection function plotted in the Fig. 11. The count rate therefore must be proportional a) to the distribution function since the velocity and mass of the particles determine whether its energy is above the detector threshold, b) to the halo model that is a measure of the DM density at the location of the local group, to the atomic number of the detector material and to the selection function:

$$Count\ rate \propto g(A) N_{\rm halo} \iint f(p,\theta,t)\Psi(E)\sigma(E) dp^3.$$

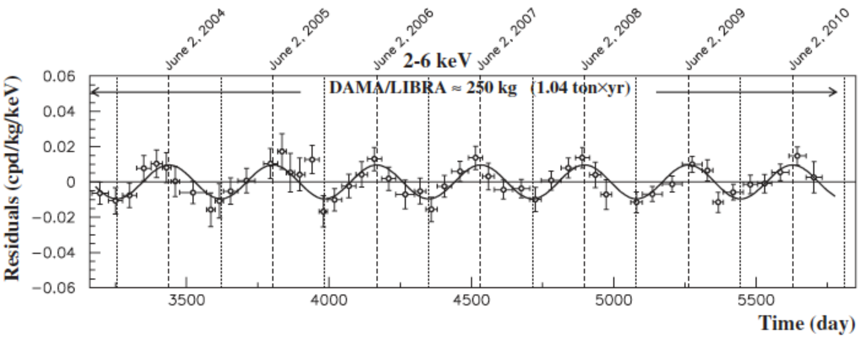

Fig. 12: Yearly fluctuation of the signal due to the motion of Earth.

For numerical details (on the theory and experiment) and a full development of the theory see the copious literature. In particular the papers by Barnabaei and her collaborators in Rome, the original paper by Drukier et al. (1986), Freese et al. (1988) and the many excellent papers published afterwards.

The data collection and improvements of the experiment continue with a significant increase of the statistics every year as shown by the results discussed in Bernabei (2016). The yearly modulation is quite evident and with the correct phase (Fig. 12). There is no doubt that a detection exists.

The experiment since the very beginning has been prepared very carefully and with a purity of the materials unmatched by any other. The Saint-Gobain (France) sodium iodide crystals went through a purification process that while expensive to buy made them unique reducing to extremely small quantities worrisome contaminants. To reduce the noise by unwanted particles the experiment has been set up in the underground INFN laboratories of the Gran Sasso and the copper used for housing the detectors was obtained from a French shipwreck that was under the Mediterranean sea since the 19th century, that is uncontaminated by the various nuclear explosions. The team did an excellent job. The detection by DAMA would suggest particles with mass in the range $1 - 10$ GeV and cross section $10^{-41} - 10^{-39}$ cm^2. It is of some interest that the CoGeNT experiment detected an unexplained event that would be in agreement with particles of mass $m_x = 9$ GeV. As reported in Feng (2010) for a WIMP mass of 100 GeV a local density of 7×10^{-25} g cm^{-3} (0.4 GeV cm^{-3}) the flux expected onto Earth is 10^5 cm^{-2}s^{-1}.

The results have been not yet duplicated by any other experiments and this is reasonably needed before the community accept the detection as evidence of WIMPS. For this reasons various scientists are skeptical, on the other hand up to now no alternative explanation has been given for the excellent and stable signal registered by DAMA.

Furthermore, and as pointed out by Bernabei, it seems meaningless to challenge the detection by the parameter space of other experiments when we do not know yet all the characteristics of the particles we are looking for. Nevertheless, considering what is at stake in such a discovery and the huge competition on this matter in the world of physics, the community would like to see more evidence. For this reason and

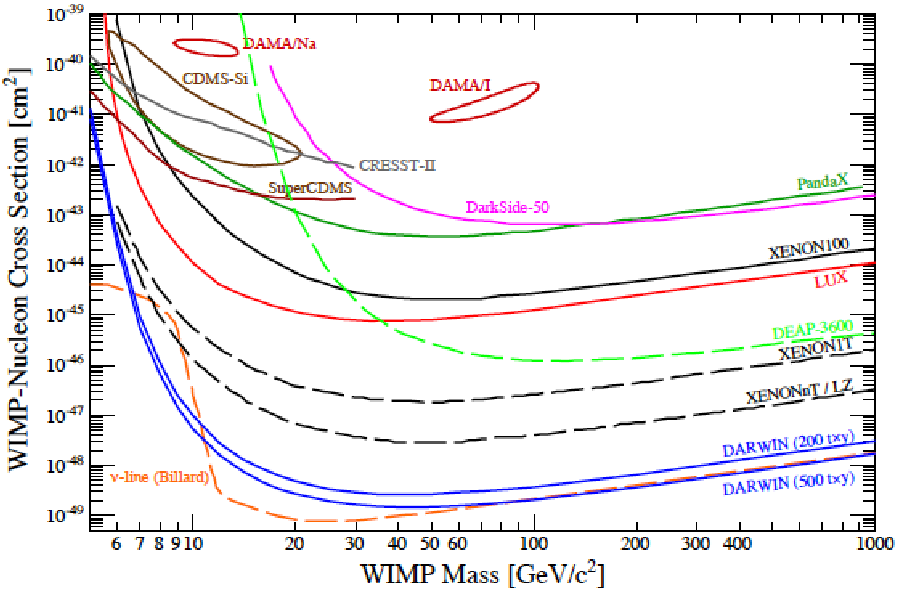

Fig. 13: Parameter space for various experiments.

thanks also to the advance of technology that allow the groups to reach the purity of the crystals of the DAMA–LYBRA (the upgraded version of the DAMA experiment) various experiments are getting ready to go using sodium iodide crystals. SABRE (Princeton) that will run in the area of Gran Sasso and also in an underground lab in Australia, DM-Ice (Yale University) that is being set in the Antarctic and ANAIS (University of Saragoza) that will run in Spain, China etc. A listing of the on going and planned experiments are in the Fig. 13 (from Laura Baudis – *Annalen der Physik*, September 2, 2015). On the horizon the largest experiment ever that, according to the planner, should give the ultimate answer about WIMPs.

Other experiments did not show anything. LUX that uses frigid liquid xenon was dismissed in May 2016 with no result. A new upgraded similar experiment, alway based on liquid xenon that should work at the Gran Sasso laboratories by 2020 may give if not a clear detection at least important guideline for the theory.

Indirect detection using the by products of annihilation or decay or going on as well improving the state fo the art of the detectors. Neutrinos are a strong possibility.

One of the problems is that WIMPs are not yet detected by the Large Hadronic Collider (LHC) at CERN and the theory tell us we should be able now to detect them or be close to it. How far could we go with the large facilities. Recently the Nobel Laureate Chen Ning Yang criticized the Chinese plan to build the Circular Electron Positron Collider (CEPC) that would be a step forward after the LHC. According to Yang the LHC confirmed the existence of the Higgs boson and however did not discover any new particle or inconsistency in the standard model of particle physics. The question is: is it worth spending 6 billion dollars? Costs and science optimization may be another problem on the horizon. Meanwhile theoreticians move to the Dark

Sector.

Among the many papers and reviews in the literature I suggest reading the work by the DAMA team, the review paper by Baudis (2016), Feng (2010) and by the Darwin team (Aalbers et al., 2016) for a clear understanding of the state of the art. After the clear, even if unattended, discovery of Zwicky we gained a lot of knowledge with a huge amount of theoretical developments and a large effort in projects aimed to detect such eluding particles. We still have a long way to go, on the other hand it is fascinating to think we are close, hopefully, to have knowledge of particles that are related to the first milliseconds of the Universe and perhaps to the beginning of Dark Matter Astronomy.

Do we need Dark Matter?

A modification of the theory (Newton's as a first step and eventually a new theory) could likely avoid the need of DM and the fact that we have not yet a certain detection (remember the DAMA data however) of the related particles legitimates the search in this direction. In 1983 Milgrom developed such a theory and not only was able to properly explain the rotation curve of galaxies without introducing the Dark Matter, but was also able to make predictions (baryonic Tully - Fisher relation for instance) that later observational work showed to be accurate. The Modified Newtonian Dynamics (MOND) is based on the concept that for very weak accelerations, say for $a \ll a_0$, where a_0 is a fundamental constant to be measured by observations, the second law of Newton's dynamic should be

$$F = ma\frac{a}{a_0}.$$

As we will see this concept and formulation has been guided by the data and we should not be surprised about the fact that a theory may break under extreme regimes. To have a smooth transition between the two regimes, $a \gg a_0$ and $a \ll a_0$ Milgrom writes the relation between acceleration and force as:

$$F = ma\mu\left(\frac{a}{a_0}\right) \text{ and acceleration of gravity as } \mu\left(\frac{g}{a_0}\right)\overline{g} = \overline{g}_{\mathrm{N}},$$

where $\overline{g}_{\mathrm{N}}$ is the Newtonian acceleration of gravity. The function $\mu(x)$ can be approximated in various ways among which the simple forms $\mu(x) = x(1+x^2)^{-1/2}$ (Milgrom, 1983), and $\mu(x) = \frac{x}{(1+x)}$ (Sanders, 2010). The difference between the two functions is rather small. The value of a_0, as derived by Begeman et al. (1991) and normalized to $H_0 = 75$ km s^{-1} Mpc^{-1}, is $a_0 = 1.2 \times 10^{-8}$ cm s^{-2}.

The difference between the Milgrom's accelerations and the Newton's acceleration is plotted in the Fig. 14 where the two acceleration have been computed as a function of the force per unit mass in the two cases.

As expected for values smaller than a_0 the two accelerations diverge and differ at the galaxy scale length. Indeed to have a feeling of the difference in acceleration at various scale lengths we compute, for various masses, the distance at which the acceleration is of the order of the critical acceleration a_0. From above we can write $F = ma\frac{\frac{a}{a_0}}{1+\frac{a}{a_0}} = \frac{GM}{r^2}$, and the distance (for $a = a_0$) $r = \sqrt{\frac{2GM}{a_0}}$, and it is plotted in the Fig. 15 where we also marked the mass of a few characteristics scale lengths. As

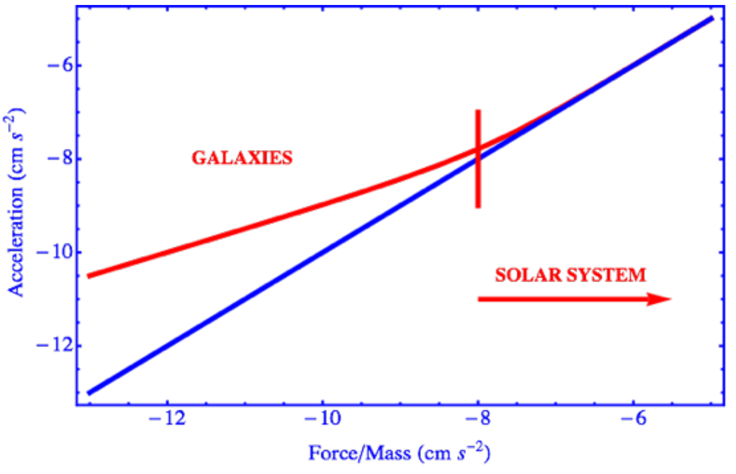

Fig. 14: Acceleration according to Newton theory (blue) and MOND (Red).

you can see, and as it has been shown above, the weak acceleration regime has to be taken into consideration with galaxies and cluster of galaxies (the red lines mark the indicative mass in solar masses at various scale lengths).

For reference on the scales the lengths of the green lines are respectively 1 pc, and 50 kpc. For a mass of 1 kg the change of regime would occur at a distance of 100 cm. For circular motion the acceleration, according to MOND in the weak acceleration regime, is $\frac{1}{a_0}\left(\frac{V^2}{r}\right)^2 = \frac{GM}{r^2}$, from which we derive $V_\infty^4 = GMa_0$. The rotation curve of any object is asymptotically flat. Following Milgrom (1983) paper about the dimensionless rotation curve of a disk (of course to fit the observations we should use a model disk + bulge of baryonic matter, for details however the reader may as well read the very clear analysis published by Milgrom) we compute the rotational velocity as a function of the distance.

We indicate with $g_{\rm N}(r) = \frac{V^2(r)}{r}\mu\left(\frac{V^2(r)}{ra_0}\right)$, the gravitational acceleration (conventional), with h a scale length characteristic of galaxies that we can use as unit of length defining $s \equiv \frac{r}{h}$, and $\nu(s) = \frac{V(sh)}{V_\infty}$. In a general way we can write $g_{\rm N}(r) = MGr^{-2}\gamma(s, t_1 \ldots t_n)$, were the function γ accounts for the distribution of mass and is a function of dimensionless parameters; $\lim\limits_{s\to\infty} \nu(s) = 1$. We also define $\xi = \left(\frac{MG}{a_0 h^2}\right)^{1/2} = \left(\frac{MG}{a_0}\right)^{1/2}\frac{a_0}{a_0 h} = \frac{V_\infty^2}{a_0 h}$, it follows

$$\frac{V^2(r)}{r}\mu\left(\frac{V^2(r)}{ra_0}\right) = g_{\rm N} = MGr^{-2}\gamma(s, t_1 \ldots t_n),$$

$$MGr^{-2}\gamma = MGs^{-2}h^{-2}\gamma = \xi^2 s^{-2}\gamma a_0,$$

$$\mu\left(\frac{V^2(r)}{ra_0}\right) = \mu\left(\frac{\nu(s)^2 s^{-1} a_0 \xi}{a_0}\right) = \mu\left(\nu(s)^2 s^{-1}\xi\right),$$

$$\nu(s)^2 s^{-1}\xi\mu\left(\nu(s)^2 s^{-1}\xi\right) = \xi^2 s^{-2}\gamma a_0,$$

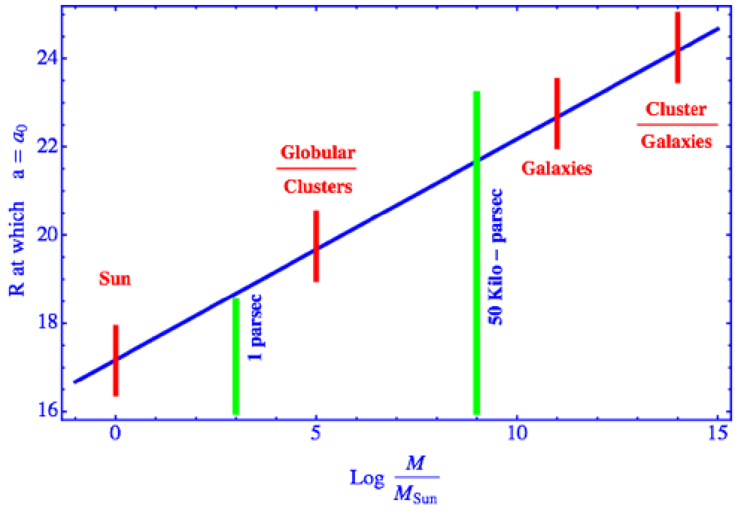

Fig. 15: The scale length at which the acceleration regime changes as a function of mass.

and using $\mu(x) = x(1+x^2)^{-1/2}$, we derive for the rotational velocity

$$\nu^8 - \gamma^2 \nu^4 s^{-2} \xi^2 - \gamma^2 = 0.$$

In the case of a disk distribution of mass (Freeman, 1970)

$$\gamma_{\rm d}(s) = \left(\frac{s^3}{2}\right) \left[I_0\left(\frac{s}{2}\right) K_0\left(\frac{s}{2}\right) - I_1\left(\frac{s}{2}\right) K_1\left(\frac{s}{2}\right) \right],$$

where I and K are Bessel functions of order 0 and 1. The rotation curves derived are shown in the Fig. 16 for different value of the parameter ξ. Analogously we can compute the contribution of the bulge, and for details and fitting of the station curves I refer to the papers by Milgrom, the work by Begeman et al. (1991) and to the excellent reviews by Milgrom (2014, 2015) and by Famaey & McGaugh (2012) among others. From Milgrom (2016), but see details also in Famaey & McGaugh (2012), we also reproduce in Fig. 17 the correlation between the total baryonic mass and the asymptotic rotational speed for disc galaxies. The yellow band is the prediction by the MOND theory for a range of a_0 values. There is a remarkable agreement! By now the theory and understanding of the observations have been largely developed also with the introduction of the composite gravitational constant, $A_0 = G\, a_0$, which replaces Newton's G. Finally it might not be a coincidence that the fundamental Milgrom's constant a_0 carries some cosmological connotation:

$$2\pi a_0 \approx cH_0 \approx c^2(\Lambda/3)^{1/2}.$$

There are also some problems, and these are very well discussed and listed in the review Famaey & McGaugh (2012), related especially to the mass of Clusters of Galaxies and to the background radiation. As far as Clusters of Galaxies are concerned either it is possible to explain the dynamics without invoking at all any form

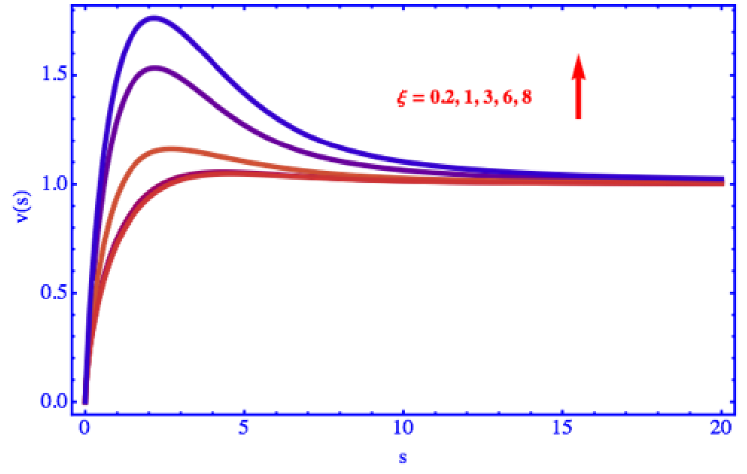

Fig. 16: Normalised rotation curves following the MOND recepit.

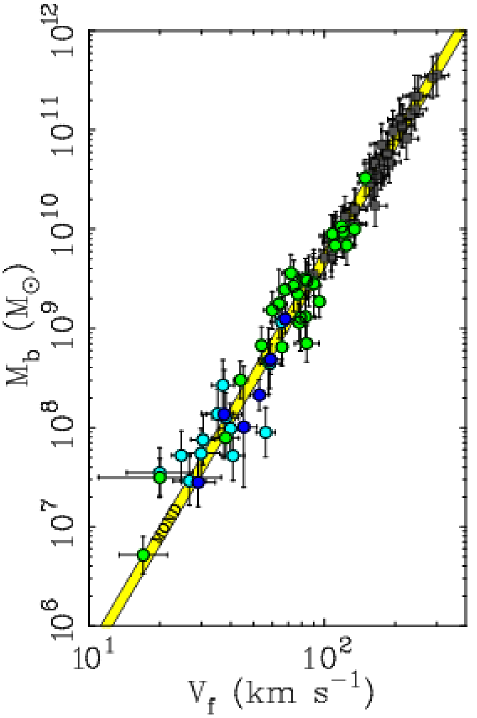

Fig. 17: Baryonic mass versus asymptotic rotational velocity (see Milgrom, 2016).

of non-baryonic Dark Matter or this may lead to a path falsifying the theory. Finally a complete theory, but this is true not only for MOND, should lead to a deep understanding at any density of matter and scale length. But it seems we are yet far from this goal. In some cases, however, it is only matter of understanding properly the observations (see for instance the recent interpretation of the Pioneer satellites anomalies)[7].

The goal here wasn't to review the MOND theory, the papers and reviews I refereed to are excellent and complete, but rather to trigger the curiosity of the students toward something different because while it is a condition *sine qua non* the knowledge of the concordance Science, it is fundamental searching for new paths and new ideas that my change completely our views and be very rewarding.

Conclusions

Each time I read and learn about the work carried out by so many capable scientists creating theories, devising and projecting experiments and working with deep insights and intelligence on the data I admire the human being with its geniality. My thoughts are led to a honest virtual world of ideas and creativity where the minds please and are pleased. Indeed the Universe as a whole may, in a sense, exist because we are—paraphrasing Cartesius, indeed—"*Cogito ergo sum; ego sum ergo mundus est*", as mentioned in the Lectures I gave in Rio de Janeiro (Chincarini, 1982).

To some extent this statement contradicts the extended Copernican principle, on the other hand it is a fact that we exist. This also leads us to the Anthropic Cosmological Principle in the sense that our location in the Universe is necessarily privileged to the extent of being compatible with our existence as observers. Following Barrow & Tipler (1986) who wrote a fascinating book on this topics, the Anthropic Principle seeks to link the parameters of the Universe to the values they should have to permit the existence of we, living beings, as observers. Further on this line according to the Strong Anthropic Principle "The Universe must have those properties which allow life to develop within it at some stage in its history". Indeed it is well known that a Universe with a different ratio of the electron proton mass, with the building of atoms different from what we know, changing the nature of the elements the way we know or their abundance would have not allowed our existence.

The Universe is very large, simple and complex and likely we needed all this complexity to have the human beings the way we know them. A smaller system would not do it. The observable universe is of the order $l = ct_0$, where t_0 is the age of the Universe, with a total mass:

$$M \sim \frac{4}{3}\pi\rho l^3 \sim \left(\text{Flat model}\, t = \frac{1}{\sqrt{6\pi G\rho(t)}}\right) \to G^{-1}t^{-2}c^3t_0^3 = G^{-1}c^3t \sim 10^5 t(s) M_\odot.$$

Assume we have a small Universe containing a single galaxy with a mass of about $10^{12}M_\odot$, from the relation above it would have expanded for 10^7 seconds (about a

[7]The satellites Poneer 10 & 11 (1972, 1973) were launched to leave the Solar system and fly in the interstellar space. After they passed the orbit of Neptune the satellites did not move as they should have done in the gravity due to the Sun (inverse square law).The anomalous acceleration was about 8.74×10^{-8} cm s^{-2} (this anomaly was immediately detected by the scientists at the Jet Propulsion Laboratory), close indeed to MOND's fundamental constant! However Turyshev et al. (2012) demonstrated that the anomaly was due to a very slight force due to heat pushing back on the spacecraft. The heat coming out from the instruments and thermoelectric power. Is it a closed issue? see ten Boom (2013).

few months) not enough to create mankind, stellar evolution etc. The mere existence of mankind requires time and a massive Universe the way we know it. But then, unless someone programmed such a thing very carefully and in all details to have this Universe, many had to be generated and this should also be explored and the eventual communication, or lack of it, be part of the theory.

"While there certainly are many things we don't understand, we do understand the Matter we're made from, and that we encounter in normal life—even if we're chemists, engineers, or astrophysicists."

Frank Wilczek. *From Physics in 100 years* (Wilczek, 2015)

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