Extended-body corrections to the inverse-square law for spherically symmetric sources

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Introduction

We attempt to explore the deviations from the standard inverse-square variation of irradiance when one considers the source as an extended-body. The ideal example to study these differences is a planet very close to its host-star. The recent advances in the study of extra-solar planets have shed light on such planets that are very close to their parent stars (Mayor et al. 1995). The irradiation received by these planets plays an important role in understanding the geological and climate properties of such planets. This underscores the importance of deriving a correct model for irradiation in such planets. The inverse-square law has been well established and has been used ubiquitously in previous studies (Kane et al. 2020). However, it doesn't suffice for extremely close planets. If one simply imagines such a close configuration, it is immediately clear that inverse-square law doesn't explain irradiation on the night-side of a close exoplanet since it assumes that the star is point-sized. This demands a new model that better explains the irradiation distribution and also incorporates the stellar limb darkening. In our approach we attempt to make a robust model that accurately predicts the irradiation distribution on such planets.

Violation of the inverse square law

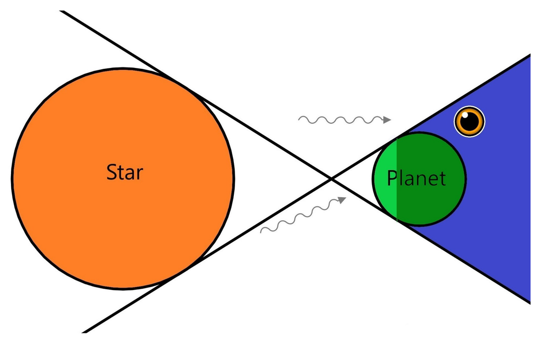

Figure 1: Boundaries of validity for the inverse-square law

Considering a star-planet system, the blue region in Fig.**1** depicts the set of all those positions from which the star is not entirely visible. An observer (represented in our case by the eye) sitting on an arbitrary blue point would see a partially eclipsed star. Since in this case, some of the photons directed to the observer are screened by the planet, the flux fails to be radial. This opens the door to possible violations of the inverse-square law on the dark green region of the planet. Quantifying the importance of the violations of the inverse-square law in the dark-green region and their dependence on the geometrical parameters of the star-planet system is the main purpose of this work.

Geometrical model

In our approach, we intend to integrate the irradiance from each surface element from the apparent surface of the star. For a close-in planet, the limits of integration are defined by the region bound between the tangents to the stellar circle from the point of observation on the planet. Additionally, the stellar surface hidden beneath the local horizon is omitted from the total flux received.

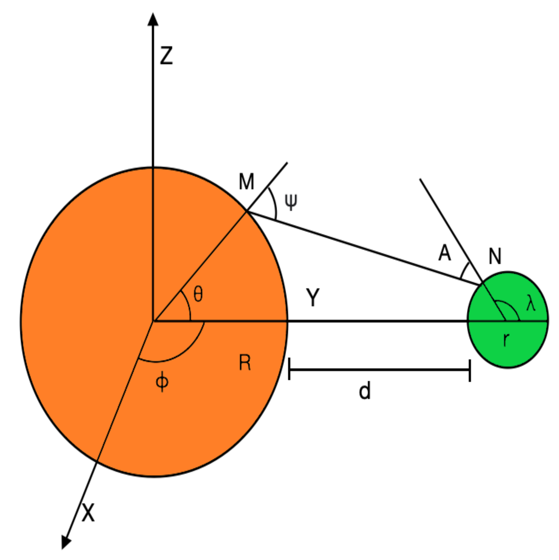

Figure 2: Star-planet system geometry in spherical coordinates

Fig.**2** shows the system geometry with λ and δ defining the latitude and longitude of the planet, respectively. d_s gives the distance between the surface element on the star to a given point on the planet, and $(r + R + d)$ is the semi-major axis(a). The star is assumed to have a bolometric luminosity L_b.

Critical point of symmetry

The position at which the light-green region of the planet (where the inverse-square law is valid) meets the dark-green region (where the inverse-square law ceases to hold) is defined as the *critical belt of symmetry* and each point of it is a *critical point of symmetry*. The latitude at which the local horizon will be a tangent to the star is given by:

$$\lambda = \cos^{-1}\left[\frac{R+r}{a}\right]$$

Without loss of generality we can conclude that all critical points on the planet that make an equivalent angle constitute a critical belt of symmetry.

Terminator angle

The terminator angle, for a given star planet system, determines how far will the terminator extend beyond the day-side of the planet. Mathematically, the terminator angle can be calculated from the slope of the exterior common tangent of the stellar and planetary circles :

$$A_t = \pi/2 + \sin^{-1}\left[\frac{R-r}{a}\right].$$

Analytical formulation

The analytical formulation of the new equation for the irradiance is given as follows

$$\int dI = \int_{-\theta_n}^{\theta_n}\int_{-\theta_n}^{\theta_n} \frac{\sigma T_0^4 R^2 \cos\theta\cos A\cos\psi(1-u(1-\mu))d\theta d\phi}{\pi (d_s)^2}$$

Here T_0 is the temperature at an optical depth of 1 and θ_n is the limit of stellar visibility that depends on a given latitude at the planet. Currently, the model assumes a linear limb darkening law, and therefore works best for Sun-like stars. The coefficient u is the bolometric linear limb darkening coefficient. Numerical integration technique is used to calculate the total irradiance.

Comparison with the inverse square law is made using the following equation:

$$I = \frac{L_b \cos i}{4\pi(a^2 + r^2 - 2ar\cos\lambda)},$$

where

$$i = \tan^{-1}\left[\frac{a\tan\lambda\cos\lambda}{a\cos\lambda - r}\right].$$

The latitude at which i becomes 90 is termed as the terminator limit for the inverse-square law since the irradiance predicted by the IS law beyond this point is zero.

Results

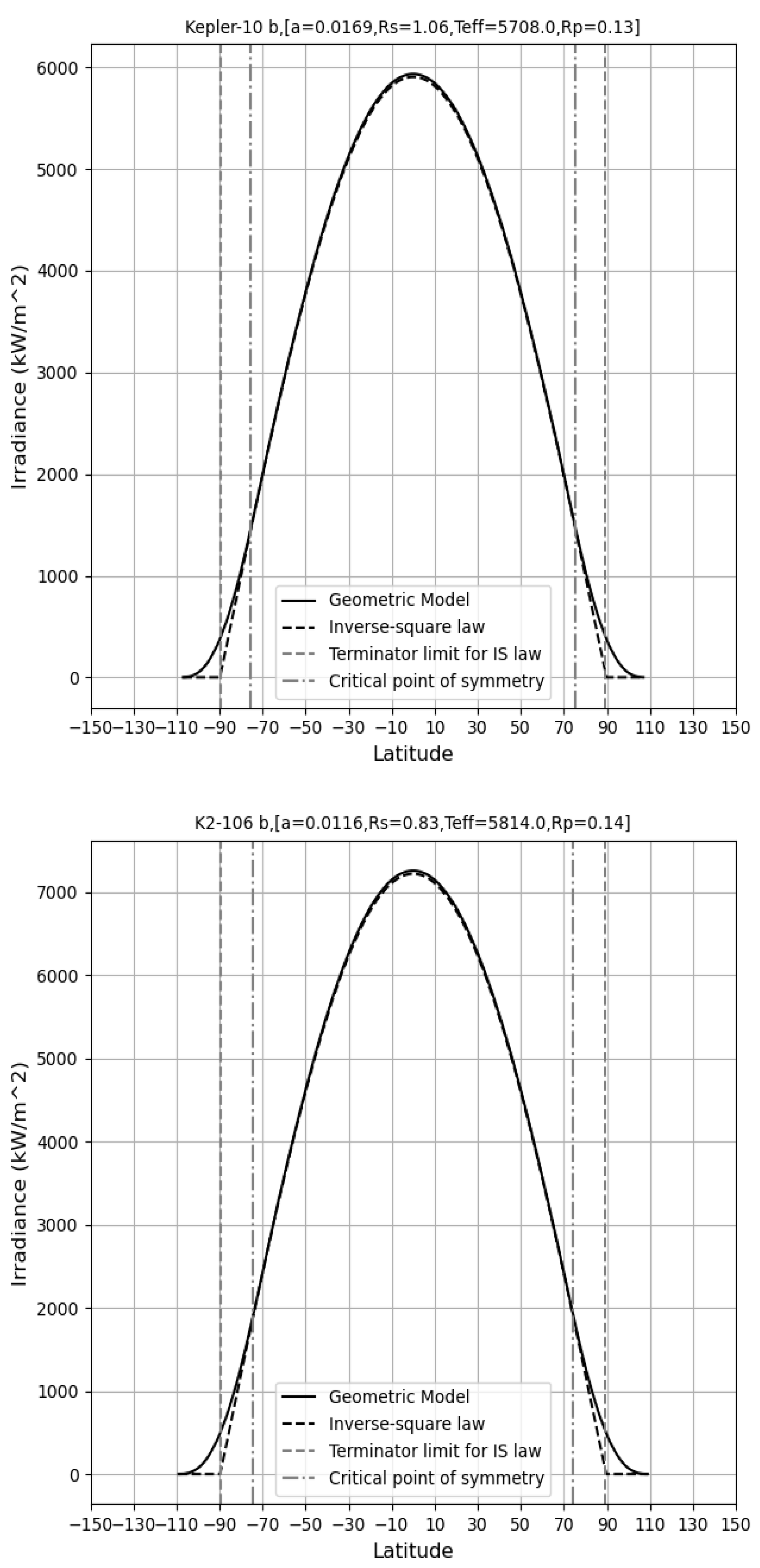

Figure 3: Irradiance plots for two Earth-like, close-in planets orbiting a solar-type host star

Analysis

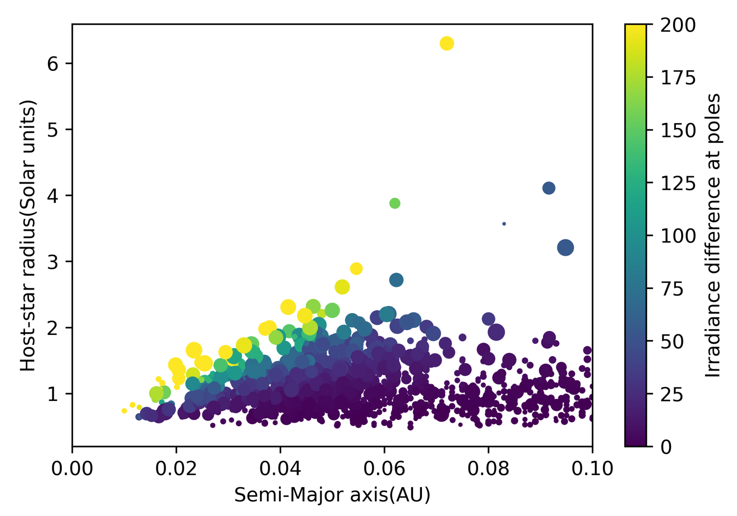

Figure 4: Difference between the two approaches for a subset of the current the exoplanet population. The size of the points indicates the radius of the exoplanet

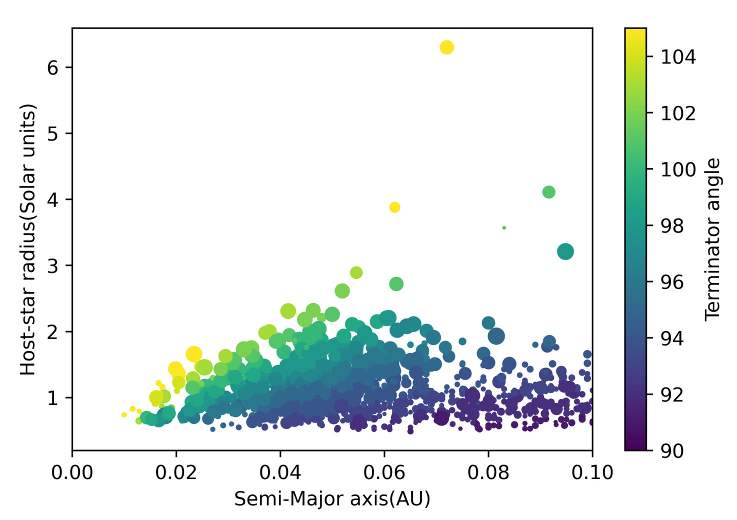

Figure 5: Terminator angle variation among the current population

An important distinction between the two approaches comes at the poles of the exoplanet. The IS law predicts zero irradiance at the poles since it follows a cosine relation of irradiance with latitude. The difference increases with increasing stellar radius and decreasing semi-major axes; This means that the relative size of the star from the planet plays a key role. Another interesting aspect of this model is the distribution of irradiation on the planetary hemisphere not facing the star. Unlike the IS model, the irradiance here extends towards the night-side, thus shifting the day-night terminator.

Discussion

It has been earlier proposed that even small changes in stellar irradiance may cause fundamental climatic changes on an Earth-sized planet, which in turn could affect its habitability (Kilic et al. 2017). The extent of difference in irradiation from the standard approach at the poles seems significant enough and is therefore likely to introduce appreciable changes in the current climate models of close-in Earth-size exoplanets. A study by Nguyen et al.(2020) reproduces similar insolation patterns and hence consolidates our results. Modifying our calculations to include the atmospheric effects remains as one of the future scopes of our work. The python code *InstellCa* uses the stellar and planetary parameters from the extra-solar planet encyclopedia (Schneider et al. 2011) and the NASA exoplanet archive (Akeson et al. 2013) to compute the irradiance plots and is openly available to the community for further research.[a]

[a] https://github.com/Mradumay137/InstellCa

References

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